SOME ESTIMATES OF THE COST OF CAPITAL TO THE ELECTRIC UTILITY INDUSTRY, 1954-57

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In its simplest form, the central normative proposition of the micro theory of capital is that the firm should adjust its capital stock until the marginal rate of return on further investment (or disinvestment) is equal to the cost of capital. Under conditions of perfect certainty—which is the assumption on which most of classical theory has been developed—the concept of the cost of capital presents no particular difficulty. It is simply the market rate of interest. Since all securities must have the same yield in equilibrium under certainty there is only one such rate per period, and it is, in principle, a directly observable magnitude. Under real-world conditions, however, we are confronted not with one, but with a bewildering variety of securities, with very different kinds and priorities of claims to portions of the (uncertain) future earnings of the firm. Since these securities will also, in general, have different anticipated yields, it is by no means clear which yield or combination of yields is the relevant cost of capital for rational investment planning. Nor, because it is based on anticipations, is the cost of capital any longer a directly observable magnitude. It must, rather, somehow be inferred from what is observable, namely, the market prices of the various kinds of claims represented by the different securities.

Although most (but not all) recent studies of investment behavior have shown some awareness of these difficulties, a common approach in empirical work has been simply to ignore the problem and to use

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without comment or explicit justification some standard index of current, nominal yields on high-grade corporate bonds (or even government bonds) as a measure of the cost of capital. Other writers use both a current bond yield series to approximate the cost of debt capital and a current profit series to measure the "availability" and hence, presumably, also the "cost" of equity capital. Still others have tried indexes of share prices, current dividend yields, or current earnings yields alone or in various averages with bond yields along lines suggested in the standard texts on corporation finance. How much error is involved in the use of such measures is still unknown, though even a cursory survey of the underlying theory suggests many grounds for apprehension on this score. But we cannot be sure. Too little work has yet been done to permit even a rough calibration of these series as proxies for the cost of capital, let alone to provide acceptable alternative series.

It is the purpose of this paper to take at least some first steps toward closing this gap in our information about the factors affecting investment. We shall attempt here to develop effective methods for inferring the cost of capital relevant for optimal investment decisions from data on the market values of securities. As a concrete application of these techniques—which we hope will prove of interest not only to economists concerned with understanding investment behavior, but also to the many corporate officials we know to be currently engaged in similar investigations—we shall present some actual estimates of the cost of capital for a sample of large electric utilities for the years 1954, 1956, and 1957.

The sample consists of 63 separate firms representing all of the (consolidated) systems classified as of 1950 as Class A by the Federal Power Commission,\(^1\) plus those of the smaller Class B systems whose assets devoted to electricity generation were at least $15 million in 1950. The cutoff at $15 million was, of course, an arbitrary one and represented what we hoped would be a workable compromise between our desire to have as large a sample as possible and yet to avoid having to find valuations for many small companies whose securities are not widely held or actively traded. Consolidated systems rather than individual companies were used to avoid the many problems (both of accounting and of valuation) posed by intercompany holdings of securities. A list of the included companies is appended as Appendix A.

As a testing ground for techniques of estimating the cost of capital the large utilities offer a number of important advantages. They permit us to have both a large sample and one in which the component firms are remarkably homogeneous in terms of product, technology, and

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\(^1\) Except for the Hartford Electric Light Company whose accounts could not be adequately reconstructed and made continuous over the entire sample period.
market conditions. In addition, as a by-product of regulation, substantially uniform accounting conventions (down to even such small, but often annoying, details as choice of fiscal year) are followed throughout the industry. In the years studied, moreover, earnings have been highly stable with few of the wide swings in year-to-year reported earnings due to strikes, cyclical or competitive shifts in demand, write-offs of assets, mergers, and the like that often render the published earnings figures virtually meaningless in more sprawling and less sheltered sectors.

As for the sample years, we are embarrassed to have to admit that 1957 was chosen because it was the latest full year available when we began this study. The year 1954, we felt, would be far enough back to show up any significant trend in the cost of capital, but not so far back as to create problems of continuity for the companies in the sample. Since there were some months of business recession in both these years we included the boom year 1956 in the sample in the hope that it might provide some evidence on cyclical swings in the cost of capital if such were sizable or important.

The paper itself will be presented in four main sections. We begin in Section I by providing an operational definition of the cost of capital and developing therefrom the link between cost of capital and market valuation (largely along lines set forth in our previous papers [14] [15] and [17]). Section II describes the econometric model and techniques to be used in estimating the cost of capital. The estimates themselves are presented along with some tests of the specification in Section III. Section IV provides an economic interpretation of the results and a comparison, insofar as that is possible, with other, more conventional estimates.

I. Valuation and the Cost of Capital

As used throughout, the term cost of capital, \( C \), will be taken to mean that minimum prospective rate of yield that a proposed investment in real assets must offer to be just worthwhile undertaking from the standpoint of the current owners of the firm. Under conditions of perfect capital markets (and even in some special cases where systematic imperfections are present) there is a one-for-one correspondence between “worthwhileness” in the above sense and the current market value of the owners’ interest. If the management of the firm takes as its working criterion for investment (and other) decisions “maximize the market value of the shares held by the current owners of the firm,” then it can be shown (see, e.g., Hirshleifer [8]) that this policy is also equivalent to maximizing the economic welfare or utility of the owners. Thus, valuation and the cost of capital are intimately related.
A. The Simple Certainty Model

The precise relation between them is most easily seen in the context of a simple certainty model in which all real assets are assumed to yield uniform, sure income streams in perpetuity and in which the market rate of interest, $r$, is given and constant over time. If, in addition, we assume perfect capital markets, rational investor behavior, and no tax differentials on different sources of income, then it can readily be shown that the equilibrium current market value, $V$, of any firm (i.e., the sum of the market values of all securities or other claims to its future earnings) is given by

$$V = \frac{1}{r} X,$$

where $X$ is the (uniform) income per period generated in perpetuity by the assets presently held. The term $1/r$ in (1)—the reciprocal of the interest rate—is commonly referred to as the market "capitalization rate" for sure streams since it represents the factor the market applies to a unit income flow in converting it to a capital stock.

For an expansion of real assets to be worth undertaking from the standpoint of the current owners of such a firm, the investment must lead to an increase in the market value of their holdings. If we let $dA$ = the purchase cost of the assets acquired, $dS^o$ = the change in value of the holdings of the original owners, and $dS^a$ = the market value of the additional securities issued to finance the investment, then differentiating (1) with respect to $A$ yields:

$$\frac{dV}{dA} = \frac{dS^o}{dA} + \frac{dS^a}{dA} = \frac{dS^o}{dA} + 1 = \frac{dX}{dA} \frac{1}{r}.$$

It follows that the cost of capital, $C$, must be the reciprocal of the market capitalization rate for earnings since from (2), $dS^o/dA \geq 0$ if, and only if, $dX/dA \geq r$, i.e., if, and only if, the rate of return on the new investment is equal to or greater than the market rate of interest.2

B. Extension to the Case of Uncertainty

When we turn from a world of certainty to one of uncertainty, the problem of defining the cost of capital in operational terms becomes a much more formidable one for which no completely general solution is yet available. We have at least been able to show, however (see [15]),

2 We have here stated (and shall continue to state) the conditions for optimality of investment decisions in terms of the rate of return or internal yield on investment. Although it is well known that there may be cases in which such a rate of return cannot be adequately or unambiguously defined (see, e.g., Hirshleifer [8]), such cases are largely ruled out by our additional simplifying assumptions.
that if we retain the assumptions of perpetual streams, rational investor behavior, perfect markets, no taxes (and no "growth" in a sense to be more precisely defined later), then an analog of the certainty valuation formula (1) does carry through to the case of uncertainty. In particular, if we restrict attention to what we have called a "risk equivalent class" of firms, then the equilibrium market valuation of any firm in such a class can be expressed as

\[
V = \frac{1}{\rho_k} \bar{X} \quad \text{for all firms in class } k
\]

where \( V \) is the sum of the market values of all the firm's securities; \( \bar{X} \) is the expected level of average annual earnings generated by the assets it currently holds; and where \( 1/\rho_k \) can be interpreted as the market's capitalization rate for the expected value of uncertain, pure equity earnings streams of the type characteristic of class \( k \).² Hence, by a straightforward extension of the reasoning in the previous section, the cost of capital for a proposed expansion in scale by any firm in the class is simply \( \rho_k \).³ The precise value of \( \rho_k \) will, of course, be different from

³ At the level of pure theory a "risk-equivalent class" can be defined in precise terms as a collection of firms such that the elements of the (uncertain) future earnings stream of each firm is proportional to (and hence perfectly correlated with) those of every other member of the class. (See [15].) For empirical purposes, however, the best that can usually be done is to work with reasonably homogeneous industries and hope that the differences among firms are small, random and not strongly correlated with any of the explanatory variables. As noted above, the utility industry probably has the best prospects for meeting these tests and that is one of the reasons why we chose it. It is perhaps also worth noting in this connection that a recent study of comovements among security prices by King [10a] does find strong (but, of course, far from perfect) correlation of price movements within most of the two-digit industries studied including the electric utility industry.

⁴ The fact that under uncertainty we have to allow, in principle, for financing by many different types of securities creates no very serious difficulty. To illustrate, let \( dS^o = \) the value of any new shares floated and \( dD = \) the value of any additional debt floated. Then from (3)

\[
\frac{dV}{dA} = \frac{dS^o}{dA} + \frac{dD}{dA} = \frac{d\bar{X}}{dA} \frac{1}{\rho_k},
\]

and it is easily seen that

\[
\frac{dS^o}{dA} \geq 0 \quad \text{requires} \quad \frac{d\bar{X}}{dA} \geq \rho_k
\]

regardless of whether the new investment is financed entirely by stock

\[
\left( \frac{dS^o}{dA} = 1 \quad \text{and} \quad \frac{dD}{dA} = 0 \right)
\]

or by bonds

\[
\left( \frac{dD}{dA} = 1 \quad \text{and} \quad \frac{dS^o}{dA} = 0 \right)
\]

or by any combination of the two. Note that for simplicity we here and throughout ignore the possible second-order repercussions of the financing method used on the market value of any already outstanding debt.
class to class, presumably increasing with the market’s uncertainty as to the level of future long-run earnings in the class (and reflecting also the nature and extent of the covariation with the returns in other classes). Though the various $\rho$’s themselves would not be directly observable they could, in principle, be inferred from the market valuations, e.g., by regressing the observed $V$ on estimates of $X$ over a cross section of firms within a class.\footnote{Although $\rho_b$ is a constant over the class, it need not, of course, be regarded as a constant over time. Variations in $\rho_b$ over time will create no serious additional problems with the interpretation of (3) as a perpetuity formula so long as such changes in $\rho_b$ as occur are in the nature of “unanticipated” changes (as would be the case, for example, if the process governing the behavior of $\Delta \rho$ over time were essentially a random walk with zero mean). Note also that the assumption that all assets generate perpetual streams is less restrictive than might appear at first sight. The perpetuity valuation formulas may still be good approximations at the level of the firm as a whole even where the component assets have finite lives, provided that the firm is close to a steady state in which replacements balance depreciation and provided $X$ is interpreted as net earnings after depreciation (but before interest).}

An important implication of (3) is that the market value of a firm depends only on its real earning power and on the market capitalization rate for pure equity streams of its class, and not at all upon the particular mix of security types that characterize its financial structure. This independence of value and financial structure is basically a reflection of the assumption of perfect capital markets—an assumption implying, among other things, that for comparable collateral, the supply curve of borrowed funds for individuals is the same as that for corporations. Hence if corporations making heavy use of borrowed funds should sell, say, at a premium relative to unlevered corporations in the same class, rational investors could always obtain a more efficient portfolio by selling the “overvalued” levered shares, purchasing the “undervalued” unlevered shares, and restoring the previous degree of leverage by borrowing against the shares on personal account. And conversely if levered shares should sell at a discount, in which case the “arbitrage” operation involves selling the unlevered shares, buying the levered shares, and unlevering them by also buying a pro rata share of the firm’s debts.\footnote{A fuller account of the arbitrage mechanism and proof of the independence proposition is given in our \[15\]. It is perhaps worth noting that the independence proposition can be proved under assumptions much weaker than those necessary to develop equation (3). In particular, neither the perpetuity assumption nor the concept of a risk equivalent class is essential (see, e.g., the discussion in our \[14, pp. 429–30\] and also Hirshleifer \[9\]).}

With reference to the cost of capital, the independence of market value and financial policy implies, of course, that the cost of capital relevant for investment decisions is also independent of how the investment is to be financed, even though the particular securities considered may, and in general will, have very different expected yields. This seeming paradox disappears as soon as it is recognized that the
independence property also requires that the common shares in levered corporations have higher expected yields than those of less levered corporations in the same class—a differential which can be thought of as compensation for the greater "riskiness" attaching to levered shares. Thus, the apparent gain in terms of the cost of capital coming from the ability of a firm to finance an investment with "cheap" debt capital is offset (and with rational behavior in a perfect capital market exactly offset) by the correspondingly higher cost of equity capital.\footnote{References here and elsewhere in the paper to differences in "risk" in relation to differences in expected yields as among securities are, of course, entirely heuristic and intended solely to provide a rationalization of the theoretical results in a way that accords with ordinary, common-sense notions about relative valuations. The results themselves, however, are in no way dependent on any special assumptions with respect to risk or "risk aversion"; and readers who find our rationalization in terms of these categories unsatisfactory are free to substitute their own.}

The reason for this effect can perhaps be seen somewhat more readily by deriving from (3) an explicit expression for the value of the shares in terms of net profits and financial structure. Making use of the identities $V=S+D+P$ (where $S =$ the market value of the common stock, $D =$ the market value of the firm's debts, and $P =$ the market value of the preferred stock) and $\bar{X}=\bar{\pi}+\bar{R}+\bar{PDV}$ (where $\bar{\pi} =$ expected net profits to the common shareholders, $\bar{R} =$ expected interest payments and $\bar{PDV} =$ expected preferred dividends) it is easily seen that (3) implies

\begin{equation}
S = \frac{1}{\rho_k} \bar{\pi} - \left( D - \frac{\bar{R}}{\rho_k} \right) - \left( P - \frac{\bar{PDV}}{\rho_k} \right).
\end{equation}

The market value of the debt, $D$, represents the capitalized value of the expected interest stream $\bar{R}$; and since the interest component of total expected earnings is less "risky" than the stream of earnings itself (at least over the range of leverage that safety-oriented creditors normally permit), it will tend to be capitalized more favorably than the rate $1/\rho_k$. Hence $(D - (\bar{R}/\rho_k))$ and $(P - (\bar{PDV}/\rho_k))$ are positive and may be thought of as in the nature of risk discounts to be subtracted from capitalized net profits in arriving at the value of the shares.

Note finally that (4) could serve equally well, in principle, with (3) as a cross-sectional valuation equation from which to infer $1/\rho_k$. Clearly, however, (3) is more efficient (since it requires estimating only a single coefficient) and accordingly will be used here as the basis for estimation.

C. The Effect of Corporate Income Taxes

When we extend the analysis to allow for the existence of corporate income taxes and the deductibility of interest payments, the most im-
portant change is that market value and financial structure are no longer completely independent. To see what is involved let us again denote by $1/\rho_k$ the capitalization rate in a given class for pure equity streams available to investors (i.e., streams of expected net profits after taxes in unlevered firms); by $\bar{X}$, a firm's expected total earnings, now to be taken as earnings before taxes as well as interest; and let $\tau$ be the (constant) marginal and average rate of corporate income taxation. Then the market value of an unlevered firm can be expressed as:

$$V_u = S_u = \frac{\bar{X}(1-\tau)}{\rho_k},$$

where $\bar{X}(1-\tau)$ is the unlevered firm's earnings after taxes. The value of a levered firm with $D$ of debt or other securities whose payments are tax deductible can be shown to be:

$$V = S + D + P = \frac{\bar{X}(1-\tau)}{\rho_k} + \tau D.$$

Note that in (6), the expression $\bar{X}(1-\tau)$ no longer represents the firm's earnings after taxes or any other standard accounting concept when $D$ is not zero and hence when $\bar{X}$ includes some tax-deductible interest. To avoid confusion, therefore, we shall hereafter refer to $\bar{X}(1-\tau)$ as "tax adjusted" earnings, using the symbol $\bar{X}^r$ for earnings after taxes in the ordinary accounting sense (i.e., for the sum of expected net profits after taxes, preferred dividends and interest payments as they actually come onto the market for sale to the various security purchasers).  

As to the meaning of (6), it says, in effect, that the government pays a subsidy to firms using certain sources of capital which under current law would include bonds, notes, and other firm contractual obligations of indebtedness but not preferred stocks or common stocks. The addition to the present worth of the firm occasioned by these tax savings is the corporate tax rate times the market value of the debt—the latter

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8 The relation between the various concepts can easily be established by observing that taxes paid will be $\tau(\bar{X}-\bar{R})$ so that $\bar{X} = \bar{X}^r + \tau(\bar{X}-\bar{R})$ and hence $\bar{X}(1-\tau) = \bar{X}^r - \tau\bar{R}$. In earlier versions of this paper, we made extensive use of valuation formulas based on $\bar{X}^r$ rather than $\bar{X}(1-\tau)$ largely because of what we felt was the greater intuitive appeal and familiarity of the former concept. This advantage, however, we now believe is not sufficient to outweigh the loss in the efficiency of estimation relative to the somewhat simpler valuation relations based on the tax-adjusted concept. Furthermore, the concept of tax-adjusted earnings, though not known by that name, is actually a fairly standard concept in the literature on capital budgeting. (See, e.g., [2].)

9 There is one exception to the rule that dividends on preferred stocks are not tax-deductible, and that, unfortunately, occurs in the electric utility industry. The deduction, however, is only a partial one and applies only to a limited set of issues (those outstanding as of 1942 or issued subsequently in redemption of such issues). Hence, the error involved in ignoring this complication (as we shall) is not likely to be important for present purposes.
being, of course, the present worth, as judged by the market, of the future stream of tax-deductible payments.\footnote{10}

With respect to the value of the common shares, the effect of the tax subsidy for interest is to change (4) to

$$\text{n} S = \frac{1}{\rho_k} \tilde{\pi}^r - (1 - \tau) \left( \frac{\bar{R}}{\rho_k} - \frac{P - \bar{P}_{dv}}{\rho_k} \right),$$

which can be derived from (6) by noting that $\tilde{\pi}^r = (1 - \tau)(\bar{X} - \bar{R}) - \bar{P}_{dv}$

$=$ net profits to the common stockholders after taxes. That the subtraction from $S$ due to debt financing is now reduced to the fraction $(1 - \tau)$ of its former size reflects the fact that the government’s claim on the firm’s earnings is essentially similar to that of a stockholder (see n. 10).

Hence while the government can claim $\tau$ per cent of the profits, it must also bear $\tau$ per cent of the “risk,” including the risk introduced by leverage. In the case of nondeductible preferred stock, no such profit-sharing takes place so that the subtraction due to this form of leverage retains its previous value.

Since the deductibility of interest payments thus makes the value of the firm a function of its financial policy, it must also make the required yield or cost of capital a function of financial policy. To see the precise nature of this dependence, let $dS^o$, as before, stand for the change in the market value of the shares held by the current owners of the firm; $dS^u = \text{the value of any new common shares issued}; dP = \text{the value of any new preferred stock issued};$ and $dD = \text{the value of any new tax-deductible debt issued},$ with $dS^o + dP + dD = dA.$ Then from (6) we have:

$$\frac{dV}{dA} = \frac{dS^o}{dA} + \frac{dS^u}{dA} + \frac{dP}{dA} + \frac{dD}{dA} = \frac{dS^o}{dA} + 1 = \frac{d\bar{X}(1 - \tau)}{\rho_k} + \frac{1}{\rho_k} + \tau \frac{dD}{dA}$$

\footnote{10 Since the proof is developed in detail in [7], we shall not reproduce it here. The reader may, however, easily convince himself of the correctness of (6) by means of the following parable. Suppose that we were back in the no-tax world of formulas (3) and (4) and that all stocks were owned by two families—the G or Government family owning precisely $\tau$ per cent of the shares in every corporation and the P or Private family owning the remaining $(1 - \tau)$ per cent plus all the bonds. (For simplicity, ignore preferred stock.) Then the value of the combined stockholdings of both families in a levered firm is:

$$S_L^P + S_L^G = S_L = V_L - D_L^P = \frac{\bar{X}}{\rho_k} - D_L^P.$$

The value of the purely Private family holdings of stocks and bonds in the firm must then be:

$$V_L = S_L^P + D_L^P = (1 - \tau)S_L + D_L^P = \frac{\bar{X}(1 - \tau)}{\rho_k} - (1 - \tau)D_L^P + D_L^P = \frac{\bar{X}(1 - \tau)}{\rho_k} + \tau D_L^P.$$}

For some additional discussion of this way of looking at the derivation see [18].
from which it follows that the cost of capital or required yield on a tax-adjusted basis is

\[ C = \rho_k \left( 1 - \tau \frac{dD}{dA} \right) \]

since \( dS^e/dA \geq 0 \) if, and only if, \( dX(1-\tau)/dA \) is equal to or greater than the right-hand side of (9).\(^{11}\)

In connection with (9) the two extreme cases of financing methods are of particular interest. For an investment financed entirely by equity capital—and remember that in this context equity capital includes non-deductible preferred stock—\( dD/dA \) will equal zero. Hence the required tax-adjusted yield or “marginal cost of equity capital” is \( \rho_k \). For an investment financed entirely by debt or other sources of capital whose payments are tax-deductible, \( dD/dA \) is unity, implying that the “marginal cost of debt capital” is \( \rho_k(1-\tau) \).

The term “marginal cost” has been placed in quotes to emphasize that while these extreme cases serve to illuminate the meaning of (9), neither is directly relevant for actual decision-making at the level of the firm. In companies large enough to have ready access to the capital markets, as would certainly be true of those in our sample, investment and financing decisions (including decisions to retire outstanding securities) are made continuously and largely independently over time. Since particular investment projects thus are not, and in general cannot, be linked to particular sources of financing, the relevant cost of capital to the firm must be thought of as essentially an average of the above costs of debt and equity capital with weights determined by the long-run average proportions of each in the firm’s program of future financing. If we denote this “target” proportion of debt as \( L \), then the weighted average cost of capital can be expressed as \( C = C(L) = \rho(1-\tau)L + (1-L)\rho_k \), or, more compactly as \( C = C(L) = \rho_k(1-\tau L) \), where the notation \( C(L) \) will be used when we want to emphasize that the cost of capital is a function of the target debt ratio \( L \).\(^{12}\)

\(^{11}\) Alternatively, the required yield could be stated in terms of before-tax earnings, \( X \) as \( \rho_k(1-\tau(dD/dA))/(1-\tau) \). There would, of course, be no reason to prefer one concept of earnings over the other as long as the perpetuity interpretation of the earnings stream is strictly maintained. The tax-adjusted yield, however, can be shown to be the relevant one for decision-making when the analysis is extended to admit assets with finite lives and, accordingly, will be used here throughout. (Cf. notes 5 and 8 above and also the discussion in our [17, n. 16].)

\(^{12}\) The determination of the optimal value of \( L \) for the firm involves many difficult issues for which no completely well-worked-out theoretical analysis is yet available. Suffice it to say here only that it does not follow that because debt is subsidized we must expect to find all firms in any class with the same degree of leverage and that at the maximum permitted by lenders. The subsidy to debt is at least partially offset by other costs incurred (and not adequately comprehended in simple static models of valuation of the kind here considered) as well as by possible interactions between debt and earnings. The utility industry provides a
Notice, finally, that while the definition of the cost of capital has become a good deal more complex as a result of the introduction of corporate income taxes, the problem of estimation remains essentially the same. It still involves only the estimation of a single capitalization rate—in this case $1/\rho_k$, the capitalization rate for unlevered, pure equity streams in the class. The difference between the cost of equity and debt capital introduces no new difficulties because the cost of debt capital does not depend on the market rate of interest on bonds, but only on the above capitalization rate and the tax rate. Hence $1/\rho_k$ remains a sufficient parameter both for economists seeking to explain rational investment behavior and for firms planning their investment programs on the basis of given financial policies.\(^\text{13}\)

D. Growth and Valuation\(^\text{14}\)

Up to this point, we have been focusing attention entirely on the role of current earning power and financial policy as determinants of the value of the firm. There are, of course, very many other factors that influence real-world valuations and some that may well be large enough and systematic enough to warrant incorporating them directly into the model rather than impounding them in the general disturbance term. Of these, one of the most important is "growth potential," in the sense of opportunities the firm may have to invest in real assets in the future at rates of return greater than a "normal" rate of return (i.e., greater than the cost of capital).

Clearly, translating such a concept into operational terms is a task of formidable proportions and one for which no universally applicable solution can be expected. For industries such as the electric utilities,

particularly good example of the latter since corporate income taxes are deductible in computing earnings allowed on the rate base. To the extent that tax is thus passed on, the ultimate value of the tax subsidy on interest is correspondingly reduced (i.e., the total derivative of debt on value will be less than the partial of $D$ on $V$ given $X$). Nor, finally, do firms always have a completely unrestricted choice as to their future debt policies, either because of restrictions incorporated in already outstanding debts or because, as in the case of the utilities, the regulatory authorities may impose upper (and in some cases allegedly even lower) bounds on debt levels.

\(^{13}\) The independence of the cost of equity capital (and hence also of debt capital) from the interest rate is, of course, an independence only within a partial equilibrium framework. In a general equilibrium setting, there is necessarily a very direct connection between the interest rate (which may be regarded to a first approximation as the yield on assets generating sure streams) and the various $\rho_k$ (which are the expected yields on assets generating streams of various degrees of uncertainty). But while the connection is direct (since they are mutually determined in the process of market clearing and jointly reflect such underlying factors as the level of wealth, the composition of the stock of real assets, attitudes toward risk and the like), there is no reason to believe that they must move closely together over time.

\(^{14}\) N.B. Since all of the main earnings and cost of capital concepts have now been introduced we shall hereafter, in the interests of simplicity, drop all subscripts and superscripts on the variables where there is no danger of ambiguity.
however—where the growth in earnings has been (and will presumably continue to be) reasonably steady—rough, but tolerable, approximations to growth potential can probably be obtained by exploiting the so-called constant-growth model. In particular, suppose that a firm has the opportunity to invest annually an amount equal to 100 $k$ per cent of its tax-adjusted earnings ($k \leq 1$), on which investments it will earn a tax-adjusted rate of return of $\rho^*$, greater than $C = C(L)$, its average cost of capital. (These assumptions imply, among other things, that earnings will grow at the constant rate of $k\rho^*$ per year.) And suppose further that these especially profitable opportunities are expected to persist over the next $T$ years, after which only normally profitable opportunities will be available. Then, by analogy to the solution we have derived for the certainty case (see [14, n. 15]), the current market value of the firm can be expressed as

\begin{equation}
V = \frac{1}{\rho} \bar{X}(1 - \tau) + \tau D + k\bar{X}(1 - \tau) \left[ \frac{\rho^* - C}{C(1 + C)} \right] T,
\end{equation}

where the first two terms, as before, represent the capitalized value of the current tax-adjusted earning power plus the tax benefits on debt; and the last term is the contribution to value of the future growth potential.\textsuperscript{15,16}

Despite the heroic simplifications invoked in its derivation, the above expression for growth potential is still by no means a simple one. It is the product of three separate elements: the profitability of the future opportunities as measured by the difference between $\rho^*$ and $C(L)$; the size of these opportunities $k\bar{X}(1 - \tau)$; and how long they are expected to last $T$. None of these component terms is directly observable, though some such as $k\bar{X}(1 - \tau)$ and possibly $\rho^*$ might be approximated by extra-

\textsuperscript{15} Several considerations led to our use of the above finite growth model rather than the more popular infinite growth model (see Gordon [7] or Lintner [12]) which (taxes and leverage aside) leads to the familiar valuation formula

$$V = \frac{X(1 - k)}{\rho - k\rho^*}.$$ 

The finite model does capture at least the essence of the S-shaped growth path which is encountered so frequently (and for good economic reasons) in empirical studies of firm and industry development. The infinite model, moreover, requires that $k$ be strictly less than unity and $k\rho^*$ strictly less than $\rho$ for a stable equilibrium value of the firm to exist at all; and, as a practical matter, for $k$ and $k\rho^*$ to be substantially below these limits to avoid absurdly high valuations. In our sample, however, $k$ is in the neighborhood of two in all three years and, we might add, that this is by no means an exceptional case. The value of $k$ has been quite close to unity for the corporate sector as a whole in recent years.

\textsuperscript{16} The expression for growth potential in (10) differs from that in [14, n. 15] by the inclusion of the term $(1+C)$ in the denominator. The new expression given here is the correct one, and we are grateful to H. M. Weingartner for calling our attention to the slip in the original derivation.
polating recent past experience. Accordingly, we shall, in this paper, take the simplest way out by focusing on the most tractable component \( k \bar{X} (1 - \tau) \), the level of investment opportunities, and impounding the others in its regression coefficient.\(^{17}\)

As an empirical estimate of investment opportunities, we have used in the subsequent estimating equations the quantity \( \frac{1}{5} (A_t - A_{t-5}) / A_{t-5} \cdot A_t \). That is, we have used a linear 5-year average growth rate of total assets times current assets. This particular form of average, denoted for simplicity hereafter as \( \bar{A} \), happens to yield consistently higher gross and net correlations with total value than other simple smoothings we have tried. But the differences are not large and the estimates of the other coefficients are not sensitive to the specific measure used.

E. Dividend Policy, Valuation, and the Cost of Capital

Under ideal conditions of perfect capital markets, rational investor behavior, and no tax discrimination as between sources of income, dividend policy would present no particular problem. In such a setting, we have shown [14] that, given a firm's investment policy, its dividend policy will have no effect whatever on the current market value of its shares or on its cost of capital; and that despite the impressions of some writers to the contrary (see, e.g., Lintner [11]), this conclusion is equally valid whether one is considering a world of certainty or uncertainty. Dividend policy serves to determine only the division of the stockholders' return as between current cash receipts and capital appreciation; and the division of the firm's equity financing as between retained earnings and external flotations.

The picture becomes considerably more complicated, however, as soon as we weaken the assumptions to allow for the present tax subsidy on capital gains and for the existence of substantial brokerage fees and

\(^{17}\) Needless to say, the particular approach adopted here with respect to representing the growth variable is not to be regarded as of general applicability and was chosen, at least in part, with a number of very special properties of the electric utility industry in mind. First, growth in earnings and assets in the utility industry does tend to be fairly steady from year to year and there is, in particular, no problem of large spurs in growth due to mergers or acquisitions. Hence current and recent past investment levels do provide at least a meaningful starting point for estimates of future investment. Second, because of the regulated status of the industry there is reason to believe that what has been impounded in the coefficient, namely,

\[ \left( \frac{\rho^* - C}{C(1 + C)} \right)^T, \]

is not only of the same sign, but at least roughly of the same order of magnitude for all companies. Certainly we do not have the wide dispersion in rates of return that would characterize, say, the railroad industry (where it is not unreasonable to suppose that for some lines \( \rho^* \) may even be less than \( C(L) \)) or the typical manufacturing industry whose component firms are much more exposed to competition from new products and new firms.
flotation costs. Under these conditions, a firm's dividend policy can, in general, be expected to have an effect on its market value, though the precise amount of the effect is impossible to determine a priori. Unlike the case of corporate debt, the tax subsidy to capital gains is not a constant but varies widely from investor to investor (with the subsidy actually being zero for the substantial part of the market represented by pension funds, foundations, etc., and possibly even negative for very small investors by virtue of the dividends-received credit and for corporate investors [including casualty insurance companies] by virtue of the partial exclusion of intercorporate dividends). Moreover, the tax disadvantages of high payout shares may be partly or wholly offset by other influences. Elderly or retired investors—a nontrivial segment of the market—who are decumulating (or at least not further accumulating) their wealth in the form of shares may find considerable savings in brokerage fees and other costs of portfolio adjustment in high payout as opposed to high capital gain securities. Even net accumulators may be induced to hold high payout shares despite their lower after-tax yields either for reasons of control (broadly understood to include shares held by management as an incident to employment) or for reasons of diversification.

Given this uncertainty as to the size, and to some extent even the direction of the dividend effect, the indicated course might seem to be simply to add a dividend term with unspecified coefficient to the structural equation (10) and let the sample determine its value. From such a valuation equation we could, of course, also go on to derive an extension of the cost of capital formula (9) running in terms of dividend policy as well as debt policy.18

The trouble with such an approach, however, is that if applied in straightforward fashion (as in Gordon [6] [7] or Durand [5]), the resulting estimate of the dividend coefficient will inevitably be strongly upward biased (and the key earnings coefficient correspondingly downward biased). Since the precise mechanism generating this bias will be discussed at length in a later section, we need not dwell on the matter further at this point beyond observing that the difficulty arises from the widespread practice of dividend stabilization. With current dividends based in large part on management's expectations of long-run future earnings, the dividend coefficient in the regression equation will

18 Although the procedure for deriving the marginal and average costs of capital in the dividend case is analogous to that for the leverage case, the derivation is considerably more complicated. Further difficulties arise from the fact that in the presence of such major market imperfections, maximizing market value is no longer always equivalent to maximizing the economic welfare of the owners. Since these and related problems are largely peripheral to the main concerns here, further discussion of them will be deferred to separate papers.
reflect this substantial informational content about $\bar{X}(1-\tau)$ along with the true effect, if any, of dividends per se on valuation.\textsuperscript{19}

Because of this confounding of the earnings and dividend coefficients, our approach here will be initially to omit the dividend variable entirely, and to focus on the problem of estimating the earnings coefficient (which is, of course, to be interpreted as the capitalization rate for earnings for companies following the sample average dividend policy). As we shall see, the procedures to be developed for obtaining consistent estimates of the key earnings coefficient will also permit us to go on to obtain at least order of magnitude estimates of the true dividend effect for the sample years.\textsuperscript{20}

F. Size and Valuation

All the valuation equations so far considered have been written as linear homogeneous functions of the independent variables, implying among other things that a given proportionate change in the values of all the independent variables leads to an equal proportionate change in the market value of the firm. The results of previous valuation studies (see, e.g., Gordon [7]) suggest, however, that the true market capitalization rate for the expected earnings of large firms may possibly differ systematically from that of small firms in the same industry.

As was true of the growth effect, there are a number of possible ways of incorporating this size or scale effect into the model. By far the simplest—and hence the approach we shall here adopt—is merely to add a constant term to the valuation equation.\textsuperscript{21} The resulting nonhomo-

\textsuperscript{19} Another possible source of bias arises from the accelerated depreciation provisions of the Revenue Act of 1954. As a result of the increased deductions permitted, part or all of the dividends paid by some companies in our sample (and in other industries as well) are considered for tax purposes as exceeding accumulated earned surplus and hence as being paid out of "capital." Such returns of capital are not taxable to investors—the investor merely being required to reduce the basis (i.e., "cost") of the security for purposes of computing the capital gain or loss on sale. Since the higher the payout, the higher the probability that the dividend may qualify as tax-exempt, the net effect is likely to be an upward bias in the dividend coefficient.

\textsuperscript{20} We hasten to add that omitting dividends is, again, not a procedure we would care to recommend for general application; and that like our solution for the growth problem, the approach was adopted, in part, with some of the special properties of the utility industry very definitely in mind. For one thing, payout ratios are far less variable from company to company in this industry so that our eventual estimate of the average capitalization rate for earnings will be a much more representative average than it would be for the general run of industry groupings. Moreover, the average payout ratio is quite high, so much so that utility stocks have long represented the example \textit{par excellence} of "income stocks." Since the clientele attracted to such stocks is likely to be much less sensitive to the tax subsidy to capital gains, the size of the dividend effect (and hence of the specification error involved in omitting it) is also likely to be much smaller here than elsewhere.

\textsuperscript{21} Our reasons for not wanting to introduce a continuous size variable, such as total assets, at this stage will become clear from the discussion of the errors of measurement problem in Section II, below.
geneous equation must then be interpreted as the linear approximation over the sample range to the underlying nonlinear relation, and the coefficient of the earnings variable as the (constant) marginal capitalization rate in the industry. The magnitude and direction of the scale effect would be indicated by the size and sign of the constant term. A negative constant term would mean that the average capitalization rate is less than the marginal and hence that the average capitalization rate tends to rise with increasing size of firm. A positive value for the constant term, on the other hand, would imply decreasing returns to scale in valuation.

G. Summary

Our analysis of the theory of valuation leads to the following structural equation:

\[(V - \tau D) = a_0 + a_1 \bar{X}(1 - \tau) + a_2 \Delta A + U,\]

where \(a_1\) is the marginal capitalization rate for pure equity streams in the class, and hence the key parameter for deriving the cost of capital; \(a_0\) is an intercept term whose size and sign will measure any effects of scale on valuation; \(a_2\) is a measure of the effects of growth potential on value; and \(U\) is a random-disturbance term. Notice that since the theory implies that the coefficient of the leverage variable \(D\) is equal to the marginal corporate tax rate \(\tau\), we have so constrained it in the above equation by incorporating it with the dependent variable. There remains the problem of specifying the nature and properties of the disturbance term and it is to this task that we now turn.

II. The Statistical Model

Least-squares estimates of the coefficients of (11) will be efficient and unbiased only if among other things (i) the variance of the disturbance term is a constant, independent of the size of the firm; and (ii) the disturbances are not correlated with the independent variables. Unfortunately, neither of these conditions can reasonably be expected to hold in our sample.

A. Heteroscedasticity

As for the variance of the disturbances, one would certainly suppose that the errors in a valuation equation (including errors in measuring \((V - \tau D)\)) are of the multiplicative, rather than the additive variety. And indeed, check of the simple scatter of value on measured earnings suggests that the standard deviation of the error term is approximately proportional to the size of firm. Any attempt to fit (11) directly, therefore, would be highly inefficient and in our sample (where the largest
firm is on the order of 100 times the smallest) the results would be completely dominated by a handful of giant companies.

In the present context, there are at least two approaches worth considering as possible solutions for this problem of heteroscedasticity: (i) dividing (11) through by \((V-\tau D)\) and re-expressing the structural relation in so-called "yield" form; or (ii) weighting each observation in inverse proportion to the size of the firm and hence to the size of the standard deviation of the error.\(^{22}\) The former (which was the approach we adopted in our first paper [15] and which was subsequently followed by Bages [1]) leads to the estimating equation:

\[
\frac{X(1-\tau)}{V-\tau D} = a_0' + a_0' \frac{1}{V-\tau D} + a_2' \frac{\Delta A}{V-\tau D} + u',
\]

where \(a_0' = \rho\) = the reciprocal of the capitalization rate for pure equity streams (or, equivalently, the "marginal cost of equity capital");

\[
a_0' = -a_0\rho; a_2' = -a_2\rho; \text{ and } u' = -\rho \frac{U}{V-\tau D},
\]

with \(\text{Var} (u')\) approximately a constant for all firms.\(^{23}\)

While an approach of this kind has the virtue of simplicity, it suffers from the fact that the variable \((V-\tau D)\) enters into the denominator of the ratios on both sides of the equation. This is not only somewhat unesthetic—since we are, in effect, using \(V\) to explain \(V\)—but will lead to biased estimates to the extent that \((V-\tau D)\) contains stochastic

\(^{22}\) A third possibility, of course, would be to use the logarithms rather than the natural values of the variables, but this would be inappropriate since the growth term, as we have here derived it, enters as an additive rather than a multiplicative or exponential term. It could be argued, of course, that given the heroic assumptions underlying that growth variable, the additional specification error introduced by the log transform is not likely to be of overwhelming concern and there is certainly some merit in this position. The trouble is, however, that our interest here is both in estimating the cost of capital and in testing certain hypotheses about valuation. For the latter purpose, as we shall see, additional variables must be incorporated into the equations and these very definitely enter in linear form. Since the other methods for dealing with heteroscedasticity are entirely adequate for that purpose and also carry through more naturally to the testing problem, we shall work with them exclusively in the sections to follow.

\(^{23}\) An alternative form of yield equation, though somewhat less convenient, would be

\[
\frac{X(1-\tau)}{V} = a''(1-\tau)^{\frac{D}{V}} + a_0'' \frac{1}{V} + a_2'' \frac{\Delta A}{V} + u'',
\]

where \(X(1-\tau)/V\) can be interpreted as the tax-adjusted earnings yield on total value (approximately the "average cost of capital" of the finance literature) and \(a_0'' = \rho\). Note that considerations of efficiency suggest using the quantity \((1-\tau D/V)\) as the leverage variable and suppressing the constant term rather than estimating separately a constant term and a coefficient for \(D/V\) as is usually done when (13) or variants of it are fitted to the data.

The economic meaning of yields such as \(X(1-\tau)/V\) and \(X(1-\tau)/(V-\tau D)\) and their relations to the cost of capital will be considered at length in Section IV, below.
elements independent of those in the numerator of the ratios. In the present case, this will mean that the coefficients of the growth and size variables will be too high (i.e., less negative) and that the estimate of the cost of capital (from the intercept term $a_0$) will be correspondingly too low. Since $(V - \tau D)$ certainly does have a stochastic component—impounded in the term $U$ in (11)—and since we have, at this stage, no basis for judging how large the resulting bias really is, we obviously cannot afford to rely exclusively on estimating equations of this form.\footnote{The problem of bias in the yield-form tests is further considered in note 28, below.} Instead, we shall use them here essentially only for checking the reasonability of the estimates obtained via the weighted regression approach.

As for implementing the weighted regression approach, the fact that the standard deviation of the error term in (11) is roughly proportional to size of firm means that the required weighting can be effected by the relatively simple expedient of deflating each of the variables by some scale variable such as the book value of total assets, denoted by $A$. Our reason for using total assets as a deflator rather than, say, total sales (as, e.g., in Neilsen [19]) is mainly that in the utility industry at least such deflated terms as $V/A$, $D/A$, or $\bar{X}(1-\tau)/A$ have natural and useful economic interpretations in their own right. The equation to be fitted, then, will be of the form

$$\frac{V - \tau D}{A} = a_0 \frac{1}{A} + a_1 \frac{\bar{X}(1-\tau)}{A} + a_2 \frac{\Delta A}{A} + u,$$

with $u = U/A$ and $\text{Var}(u) = \text{a constant}$.

One question that immediately arises in connection with (14) is the status of the constant term. Recall that we are interpreting the basic valuation equation (11) in the original, undeflated variables as a linear approximation over the sample range, with its constant term $a_0$ serving as a measure of the effect of scale on valuation. To preserve this interpretation we must, therefore, regard the derived deflated regression (14) as homogeneous, that is, as being fitted with no constant term and with the coefficient of the variable $1/A$ now measuring the size effect. Failure to suppress the constant term in the deflated regression would imply that the book value of total assets appears along with expected future earnings as one of the explanatory variables in the original equation—a specification that makes little sense from the standpoint of the theory of valuation. We shall, of course, attempt to check the validity of the assumption that the regression in deflated form has a zero constant term, but there are complications involved, as will become clear from the discussion in the next section.
B. Errors of Measurement in Expected Earnings

In using an equation such as (14) to estimate the cost of capital, the key variable is, of course, $\bar{X}(1-\tau)/A$, defined, it will be recalled, as the market’s expectation of the long-run, future, tax-adjusted earning power of the assets currently held by the firm. Since it is an expectation, it is not directly observable or measurable and the best that can normally be done is somehow to approximate it from the firm’s published accounting statements. This best, unfortunately, is likely to be none too good even in an industry such as the electric utility industry where there is substantial uniformity of accounting conventions among firms. Where there are, at least in our sample period, no firms suffering net losses; and where large, year-to-year random fluctuations in reported earnings seem to be relatively rare.

The implications of these inevitable errors in the measurement of earnings for the problems at hand are perhaps most easily seen by expressing the underlying structure in the following system of equations where, to simplify the notation we let

$$V^* = \frac{V - \tau D}{A} ; X^* = \frac{\bar{X}(1-\tau)}{A} = \text{the “true” unobservable expected earnings;}$$

$X =$ deflated earnings as measured from the accounting statements; and $Z_i, i=1 \cdots m$, stand for all other relevant variables (including constants, where appropriate):

$$V^* = \alpha X^* + \sum_{i=1}^{m} \beta_i Z_i + u$$

(1)

$$X = X^* + v$$

(2)

$$X^* = \sum_{i=1}^{m} \gamma_i Z_i + w,$$

(3)

where some $\gamma_i$ and $\beta_i$ may be zero; and with the error terms assumed to be independent of each other and to have mean zero and (constant) variances $\sigma_u^2, \sigma_v^2,$ and $\sigma_w^2$, respectively. In words, the value of the firm depends on expected earnings and certain additional explanatory variables; measured earnings are merely an approximation to true expected earnings, the error of measurement being $v$; and lastly, some, at least, of the explanatory variables are also correlated with (and hence convey information about) the true, but unobservable $X^*$.\footnote{Note that we have assumed $X$ to be an unbiased estimate $X^*$. We shall also assume that the error of measurement $v$ is uncorrelated with any of the $Z_i$ ($u$ and $w$ meeting this requirement by construction). Neither of these assumptions is essential for present purposes, though they simplify the presentation, and we shall consider later some of the implications of relaxing them.}
Equations (15, 1–3) thus constitute a simultaneous system in which \( V^* \) and \( X \) are, in effect, endogenous variables, and the \( Z_i \)'s are exogenous variables. It follows then that if we attempt to fit by direct least squares the single equation,

\[
V^* = aX + \sum_{i=1}^{m} b_i Z_i + \epsilon',
\]

in which \( V^* \) is regressed on the \( Z_i \)'s and the endogenous, measured earnings \( X \), the error term \( \epsilon' \) will not be independent of \( X \), and the coefficients of (16) will be subject to the equivalent of the familiar simultaneous equations bias. More concretely, it can readily be shown (see, e.g., Chow [3, esp. pp. 94–98]) that in the limit for large samples the coefficient of \( X \) will be given by

\[
a = \alpha \frac{\sigma_\epsilon^2}{\sigma_w^2 + \sigma_v^2},
\]

which is less than the true value \( \alpha \), and the more so, the larger the variance of the error of measurement \( \sigma_v^2 \), and the better the included exogenous variables are as proxies for earnings (i.e., the smaller the value of \( \sigma_w^2 \)).

As for the other variables, the coefficients will be given by

\[
b_i = \beta_i + \gamma_i \alpha \frac{\sigma_v^2}{\sigma_w^2 + \sigma_v^2} = \beta_i + \gamma_i [\alpha - a]
\]

and thus may be larger or smaller than their true values \( \beta_i \), depending on the direction of correlation with \( X^* \) (i.e., on the sign of \( \gamma_i \)).

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28 The above expression for the bias in the earnings coefficient was derived on the assumption that (16) was fitted with a constant term. If the equation were fitted without a constant term (and if, as we have assumed, there is no constant term in the true specification), then the apparent bias would be considerably smaller. The reason is, of course, that the bias, by flattening the slope of the regression, tends to produce a positive intercept even where none really belongs. Hence forcing the regression through the origin and eliminating the artificial intercept offsets some of the distortion. The offset is only partial, however, and forcing the regression through the origin cannot be regarded as a satisfactory substitute for the more elaborate methods for eliminating the bias to be introduced below.

27 Note that even if some \( \beta_i = 0 \) (implying that the corresponding \( Z_i \) really has no effect on market value), its estimate \( b_i \) might still be positive if \( \gamma_i > 0 \), and that \( b_i \) might be quite large if \( \gamma_i \) is large and if the measurement error in \( X \) is substantial (so that \( a \) is considerably smaller than \( \alpha \)). This is, of course, precisely the "information effect" or proxy variable bias we were concerned about in connection with the dividend variable (cf. above, Section II.E and see also the discussion in our [16]).

28 In the light of the system (15) it is interesting to re-examine the problem of bias in the yield-form equations (cf. Sec. II.A). Using the notation above, but interpreting the variables as undeflated, the proposed yield equation becomes
C. An Instrumental Variable Approach

Recasting the original structure in the form (15) not only serves to clarify the nature of the biases introduced by errors of measurement, but also to suggest a remedy, namely an instrumental variable approach. For reasons of computational simplicity as well as ease of interpretation in the present context, we shall implement this approach by means of a two-stage procedure formally equivalent to the two-stage least-squares method of Theil [21]. See also [13]. Operationally, this means first regressing the endogenous variable \( X \) on all the instrumental variables \( Z_i \), thereby obtaining estimates \( g_i \) of the coefficients \( \gamma_i \) in (15.3). From these estimates a new variable \( \hat{X} \) is formed, defined as

\[
\sum_{i=1}^{m} g_i Z_i,
\]

and thus constituting an estimate of \( X^* \) from which, if our assumptions are correct, the error of measurement \( \nu \) will have been purged.\(^{20}\) If \( \hat{X} \)

\[
\frac{X}{V^*} = \frac{1}{\alpha} - \frac{1}{\alpha} \sum_{i=1}^{m} \beta_i \frac{Z_i}{V^*} + u',
\]

where

\[
\nu = \frac{1}{\alpha} \frac{u}{V^*},
\]

Under our assumptions (a) that \( X \) is an unbiased estimate of \( X^* \), and (b) that \( \nu \) is uncorrelated with the \( Z_i \), the measurement error \( \nu \) leads to inefficiency, but not to bias since that error is independent of the explanatory variables. The trouble comes rather from the fact that the endogenous variable \( V^* \) is now included among the explanatory variables, thereby leading to a lack of independence between the error term \( u' \) and the variables \( Z_i/V^* \). Thus, the nature of the bias is different when the form of the equation is changed; but given the fact that we have two endogenous variables, bias in one form or another is bound to arise under any single-equation approach.

\(^{20}\) That is, (15.2) and (15.3) imply

\[
X = \sum_{i=1}^{m} \gamma_i Z_i + v + w
\]

so that

\[
\hat{X} = \sum_{i=1}^{m} \gamma_i Z_i = X - v - w = X^* + v - v - w = X^* - w,
\]

the \( v \) term being eliminated. The additional error term \(-w\) causes no problems (other than loss of efficiency) since by construction \( w \) is orthogonal to all the \( Z_i \) and hence to \( \hat{X} \). In practice, of course, we do not know the \( \gamma_i \) exactly but only the regression estimates of them, \( g_i \). Hence, in defining \( \hat{X} \) as \( \sum_{i=1}^{m} g_i Z_i \), there is another error term of the form \( \sum_{i=1}^{m} (\gamma_i - g_i) Z_i \), which will, however, tend to approach zero as the sample size increases.
is then used in the second stage as the earnings variable in (16) in place of $X$, (and if the conditions for identification are met) the resulting estimates $a$ and the $\beta_i$ can be shown to be consistent estimates of $\alpha$ and the $\beta_i$ in the basic structural equation (15.1).

As for the specific exogenous or instrumental variables to be used we have already considered two, growth and size. In addition, if the coefficients in (16) are to be identified, we must find at least one more such variable that may properly be included in the first-stage regression of $X$ on the $Z$, and properly be excluded from the second-stage equation (16). We say at least one more because while even a single such variable will suffice for identification, additional variables, by actually overidentifying (16), will help to reduce the loss in the efficiency of estimation—that loss being the price we normally have to pay for gaining consistency by using an instrumental variable approach. In concrete terms, what we need are, of course, variables that can be expected to be strongly correlated with (and hence to convey considerable information about) $X^*$ and, at the same time, to be uncorrelated with the error of measurement in $X$.\textsuperscript{30} Fortunately, in the present context, we have a number of likely candidates readily at hand.

The obvious first choice as an instrumental variable for long-run expected earnings is, of course, total assets. This will not provide a perfect measure since the rate base for regulation is not quite the same as total assets and since the allowable rate of return on the base varies somewhat from state to state. Given the fact of regulation, however, and the (partly consequent) substantial uniformity of accounting conventions in the industry, the amount of information conveyed by total assets is still substantial, and certainly far more so here than for virtually any other industry. Note, incidentally, that the fact that all our variables are deflated by total assets creates no particular problem. We can, in effect, incorporate total assets in the first-stage regression by the simple expedient of fitting it with a constant term.

Another presumably very powerful instrumental variable for the net profit component of total earnings would be current dividends paid. As has been stressed here at several points, the practice of dividend stabilization—and the utility industry is the \textit{locus classicus} of such policies—means that dividends and dividend changes indirectly convey a considerable amount of information at least about management's expectations of long-run future profits. In fact, it will be recalled, it was precisely because they conveyed so much information that we were led to omit dividends entirely from the basic valuation equation in order to

\textsuperscript{30} The second requirement is important, of course, because to the extent that $v$ is correlated with the $Z$, part of the measurement error will remain in $\hat{X}$. In fact, if the correlation is very strong, $\hat{X}$ will be little different from and hence little better than $X$ itself.
avoid hopeless confounding of the real and the purely informational effects. The instrumental variable approach thus permits us to break this impasse and to salvage at least some of the informational content of dividends.\footnote{It may seem strange at first glance to treat dividends as an exogenous variable in an equation "explaining" $X^*$ when the causation clearly runs from $X^*$ to dividends as a matter of business policy. The fact that dividends may be an endogenous variable from the standpoint of some larger system creates no serious problem at this point, however, since the only use here made of the coefficients in the first stage is in building up an $\hat{X}$ from which the measurement errors in $X$ have been purged. As long as this is accomplished by the first-stage variables taken as a set, the individual coefficients are of little moment. Eventually, of course, when we attempt to put bounds on the effects of dividends on value, we shall have to face up to the endogenous nature of dividends. See below, Section III.E.2.}

For the interest component of expected earnings, it might be thought that the reported figure would be sufficiently close to the true value so that no specific instrumental variable need be introduced on this account. A glance at the accounting statements, however, will show that even interest paid is not an unambiguous concept in the utility industry (nor are the accounting practices completely uniform from company to company, particularly in such matters as the treatment of interest on construction and the separate reporting of interest on short-term obligations). There is the further difficulty that some companies may have issued (or retired) bonds during the year with the result that the reported interest payments for the year represent less than a full-year’s interest obligation on the debt outstanding at the end of the year (or more than the future level of obligations in the event of a retirement).

Accordingly, an instrumental variable may well prove worthwhile for interest and preferred dividends with the natural choices being the market value of the debt and of the preferred stock. Inclusion of these capital structure variables has the further advantage of picking up any systematic correlation between leverage and the level of total earnings as might arise, for example, if regulatory commissions, as has been suggested, attempt to pass the tax savings from interest deductions forward to consumers by lowering rates.

Though the variables described above would seem adequately to meet the first requirement for good instrumental variables—namely that they be correlated singly and in combination with the unobservable market expectations of normal earnings—we can, of course, never be entirely sure that they also meet the second test—namely that they be uncorrelated with the error in measured earnings. Actually, this independence condition cannot be met literally and exactly. A transient upward fluctuation in current measured profits, for example, must show up either in temporally higher dividends, or more likely in a level of total assets higher than it otherwise would have been. But while complete independence is not to be expected, we would doubt that such
correlation as does exist is so large as to dash all hopes for substantially improving the estimates by the instrumental variable procedure.

III. The Results

A. Definitions of the Variables

Before presenting and discussing the results in detail, some brief remarks may be in order with respect to the precise definitions of the variables. Measured, tax-adjusted earnings can be derived from published accounting statements either (a) by means of the formula \( \bar{X}(1-\tau) \) where \( \bar{X} \) is total before-tax earnings, and \( \tau \) is the marginal corporate tax rate of 52 per cent in the sample period; or (b) from the formula \( \bar{X}^f - \tau \bar{R} \), where \( \bar{X}^f \) is after-tax earnings, and \( \bar{R} \) is expected interest payments (cf. n. 8). If tax liabilities were exactly equal to 52 per cent of profits as reported to the shareholders, the two concepts would, of course, be identical. In practice, however, profits as defined for tax purposes do not in general coincide with those reported to the public and the consequent discrepancy between measures (a) and (b) is large enough to force a choice to be made.

It is difficult on purely a priori grounds to establish a decisive case for either alternative. Measure (b) has perhaps an advantage in that reported \( \bar{X}^f \) is the concept of earnings in terms of which the "reasonable return" is defined by the regulatory commissions. In addition, measure (b) permits an exact derivation of regression equations describing the valuation of shares from those in terms of the value of the firm as a whole—a property that is of considerable value for constructing tests of conflicting hypotheses about the influence of leverage on valuation. Since it also happens to be the case that measure (b) is consistently and substantially the better of the two by the goodness-of-fit criterion, that measure will be used here throughout, and tax-adjusted earnings will thus be estimated as \( \bar{X}^f - \tau \bar{R} \).\(^{32}\)

In building up our measure of tax-adjusted earnings from the accounting statements we have taken from *Moody's Utilities* the current-year figures for net profits, preferred dividends, and interest paid as defined by the companies themselves. Initially, when we first recognized the seriousness of the errors of measurement problem we experimented with a number of alternative concepts of earnings (e.g., adding back to profits any allocations to a reserve for deferred taxes). None of these concepts, however, appeared to be better than the actually reported current earnings in the sense of yielding consistently higher gross and

\(^{32}\) In terms of the numerical results, the main difference between the two measures is that measure (a) turns out to yield somewhat lower estimates of the cost of capital. In all other respects, however, such as the contributions of the other variables, the behavior of the systems over time and the results of various tests applied to check the specification, the two measures, insofar as we have tested them, give an essentially similar picture.
net correlations with $V$ or of showing less evidence of attenuation in
direct least-squares regressions. The same was true, moreover, for simple
equally weighted two- and five-year averages of past reported earnings.
The relatively poor performance of such averages is probably a reflection
of the fact that in the utility industry, the purely "transitory,"
random components in earnings tend to be quite small relative to the
"permanent" shocks (such as those induced by changes in regulation
policies or in accounting conventions). Where such is the case, it might
well be that some unequally weighted average with high weight on the
current year probably exists that would do better than either current
earnings alone or a simple average. We have, however, made no further
attempts to find such an average because there is no real hope that even
such an improved measure would be sufficiently error-free to permit
dispensing with the instrumental variable procedure. At best there
might be a small gain in efficiency, but the eventual second-stage esti-
mate is unlikely to be very different. For simplicity, therefore, we chose
to work with current-year earnings (though, once again, this is not a
procedure we would recommend for other industries, particularly those
subject to cyclical fluctuations in earnings).

Another troublesome variable offering several possible choices of
definition is the debt variable. Originally, we had thought we could
focus exclusively on long-term debt, ignoring short-term liabilities most
of which would presumably be noninterest bearing and which, in any
event, we expected to be small and relatively uniform throughout the
industry. The major definitional problem would then be that posed by
convertible issues selling at a substantial premium. As it turned out,
however, such issues were quite unimportant during our sample period;
but short-term liabilities proved to be both highly variable and in some
cases very large in relation to total liabilities. In part, at least, this
variation seems to stem from the practice of many companies of financ-
ing construction with short-term bank loans that are later refunded
after completion of the project. Since it was not always possible to

---

33 The main danger in using current earnings arises from the possibility that because of,
say, industry-wide strikes or cyclical disturbances, current earnings may be a systematically
biased estimate of long-run earnings throughout the industry as a whole. That is, in terms of
the system (15), the second equation would have to be replaced by $X = \alpha X^* + \epsilon, \lambda \neq 1$, implying
in turn that the ultimate second-stage estimate of $\alpha$ will be a biased one—yielding $\alpha/\lambda$ rather
than $\alpha$. Where such industry-wide distortions are present, an average of past earnings will
be a less biased measure of $X^*$ and should be used in the first stage even in the unlikely event
that it did not perform as well as current earnings in the direct least-squares regression of $V$ on
$X$. In our industry, it happens that the sample mean (deflated) current earnings are so close
to the sample mean average earnings that nothing much would be gained in using the average.
This is not to say, of course, that either current earnings or average earnings constitute an
unbiased estimate of $X^*$. We suspect that there probably is some (slight) systematic under-
statement of earnings in both cases due to the more generous depreciation allowances per-
mitted for tax purposes during the Korean War (and by the Revenue Act of 1954) and re-
lected in profits, either directly or indirectly, via the reserve for deferred taxes.

---
separate out such loans from other short-term liabilities, the safest
course seemed to be to include all short-term liabilities as well as long-
term liabilities in the debt variable. Any errors that this approach may
involve are not likely to create serious difficulties for present purposes
of estimating the cost of capital.34

Debt, preferred stock, common stock and their sum, total value of
the firm, are taken at market values and are based on price quotations
as of the end of each sample year. To iron out any purely short-term
randomness in the price quotations, a simple average of December and
January weekly closing prices was employed. The necessary price quo-
tations were always available for common and preferred stock issues, but
not always for long-term debt issues where an issue was privately placed
and held. In such cases, we estimated the market value on the basis of
the yields reported by Moody’s for comparably rated issues of equiva-
 lent term. For the short-term liabilities, we had, of course, no choice but
to use the book value of such liabilities. Our measure of \( V \), therefore,
must certainly be regarded as subject to error. But there is no reason to
believe that these errors (plus the inevitable purely clerical errors) are
of a sufficiently systematic nature to bias the estimates.

B. The Single-Equation Least-Squares Estimates

Turning now to the results themselves, we first present in Table 1
the estimates obtained from single-stage, least-squares regressions using
reported current tax-adjusted earnings as the earnings variable.35 If
our analysis has been correct, these estimates should display at least
the more obvious symptoms of attenuation bias; and a glance at the
top panel of the table shows that such is indeed the case. Here the equa-
tion has been fitted with a constant term—a specification, it will be
recalled, that is equivalent to adding total assets as an explanatory
variable in the undeflated equation. These constants are not only sig-
nificantly positive in two of the three years—on the order of three to
four times their standard errors—but are quite large relative to the
mean value of the dependent variable, which is approximately .9 in
all three years. The effect is particularly marked in 1954, where the
value for the constant of .274 would be far too high to be taken seri-
ously as a measure of the true explanatory contribution of total assets
to market value if earnings were measured without error.36

34 Even though the debt variable may include random measurement error, the fact that it
is incorporated into the dependent variable keeps such errors from introducing any systematic
bias in the coefficients of the value equation. Some problems may arise, however, where tests
of the specification are involved. See below, Section III.E.1.
35 The means and standard deviations of the major variables along with the matrix of sim-
ple correlation coefficients are presented in Appendix B.
36 Just why the measurement error might be higher in 1954 than in either of the two subse-
quenet years is still not entirely clear to us. Possible explanations might be (a) that fundamental
<table>
<thead>
<tr>
<th>Year</th>
<th>Coefficients of</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Earnings (\bar{X} - r\bar{R})/A)</td>
<td>Size (1/A \cdot 10^3)</td>
<td>Growth (\Delta \bar{A}/A)</td>
<td>Mult. (R)</td>
<td>Adjusted Standard Error</td>
<td>Ratio of Adjusted Standard Error to Mean (V/A)</td>
<td></td>
</tr>
<tr>
<td>1957</td>
<td>.164 (.06)</td>
<td>15.7 (1.2)</td>
<td>-.278 (.08)</td>
<td>1.37 (.24)</td>
<td>.88</td>
<td>.057</td>
<td>.052</td>
<td></td>
</tr>
<tr>
<td>1956</td>
<td>.057 (.06)</td>
<td>15.6 (1.2)</td>
<td>-.122 (.08)</td>
<td>.846 (.23)</td>
<td>.88</td>
<td>.057</td>
<td>.051</td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td>.274 (.06)</td>
<td>13.7 (1.3)</td>
<td>-.192 (.06)</td>
<td>.305 (.15)</td>
<td>.83</td>
<td>.054</td>
<td>.045</td>
<td></td>
</tr>
<tr>
<td>1957</td>
<td>-</td>
<td>16.0 (1.34)</td>
<td>-.277 (.08)</td>
<td>1.39 (.23)</td>
<td>.88</td>
<td>.057</td>
<td>.052</td>
<td></td>
</tr>
<tr>
<td>1956</td>
<td>-</td>
<td>16.6 (.39)</td>
<td>-.111 (.07)</td>
<td>.926 (.21)</td>
<td>.87</td>
<td>.057</td>
<td>.051</td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td>-</td>
<td>19.2 (.43)</td>
<td>-.205 (.07)</td>
<td>.466 (.17)</td>
<td>.75</td>
<td>.063</td>
<td>.053</td>
<td></td>
</tr>
</tbody>
</table>

* Constrained to equal zero.
In the bottom panel of the table, we present the results obtained with the same variables but with constant term suppressed. As would be expected (cf. n. 26), the estimates of the capitalization rate are all higher (especially so in 1954). If our analysis is correct, however, they are still somewhat too low, so that still higher values should presumably be found in the second-stage regressions.\footnote{For what it may be worth as an aid in judging goodness-of-fit, the table also shows a column labeled “Multiple $R$” for the no-constant regressions. The entry was computed according to the usual definition of $R$ (i.e., $1 - R^2 =$ the ratio of the residual variance around the regression to the variance of the dependent variable) even though this may lead to nonsense results when the fit is extremely poor (since the residual variance around the constrained regression line may actually exceed the variance of the dependent variable). An alternative and probably more meaningful way of looking at goodness-of-fit with constant suppressed is to ask how well the regression enables us to “predict” the value of the firm. For this purpose we show the adjusted standard error—the adjustment consisting of summing the squared deviations not around the actual sample mean residual but around zero which is the presumed mean if the specification were really correct—and the adjusted error as a per cent of the sample mean value of $V/A$.}

C. The First-Stage Regressions on the Instrumental Variables

Table 2 shows the first-stage regressions of reported earnings on the instrumental variables. To help in interpretation, the results are presented with and without dividends included among the set of instrumental variables. As can be seen from the first panel, total assets appear to convey considerable information about earnings in this industry—the constant terms ranging from just under 4 to just over 7 times their standard errors. Given total assets, moreover, little additional information seems to be provided by the other variables as evidenced by the low values of the multiple correlation coefficient.

The group of regressions in the second panel, which will serve as the basis for the computed earnings variable in most of the subsequent second-stage regressions, involves the full set of instrumental variables including dividends. As expected, the dividend variable makes a very substantial contribution in all three years. In fact, the combination of dividends, debt, and preferred stock now proxies for the earnings stream so well that there seems to be relatively little left for total assets to contribute (especially in 1956 where total assets are completely swamped).

As for the actual numerical values of the coefficients, there is, of course, little point in attempting to interpret them separately. The relevant concern rather must be whether they seem to behave “sensibly”
<table>
<thead>
<tr>
<th>Year</th>
<th>Constant</th>
<th>Growth $\Delta A/A$</th>
<th>Debt $D/A$</th>
<th>Preferred Stock $P/A$</th>
<th>Dividends $d/c/A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>.007</td>
<td>(.007)</td>
<td>-.002</td>
<td>(.02)</td>
<td>-.022</td>
</tr>
<tr>
<td>1956</td>
<td>.0055</td>
<td>(.009)</td>
<td>-.001</td>
<td>(.02)</td>
<td>-.01</td>
</tr>
<tr>
<td>1954</td>
<td>.0010</td>
<td>(.007)</td>
<td>-.001</td>
<td>(.02)</td>
<td>.004</td>
</tr>
<tr>
<td>1957</td>
<td>.0073</td>
<td>(.006)</td>
<td>-.006</td>
<td>(.01)</td>
<td>.029</td>
</tr>
<tr>
<td>1956</td>
<td>.0001</td>
<td>(.006)</td>
<td>.029</td>
<td>(.02)</td>
<td>.040</td>
</tr>
<tr>
<td>1954</td>
<td>.0011</td>
<td>(.005)</td>
<td>.049</td>
<td>(.02)</td>
<td>.024</td>
</tr>
</tbody>
</table>

Dependent Variable: \((\bar{Y} - \bar{D})/A\)

<table>
<thead>
<tr>
<th>Year</th>
<th>Coefficients of Mult. $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>.31</td>
</tr>
<tr>
<td>1956</td>
<td>.13</td>
</tr>
<tr>
<td>1954</td>
<td>.19</td>
</tr>
</tbody>
</table>

\(Y = \beta_0 + \beta_1 \Delta A/A + \beta_2 D/A + \beta_3 P/A + \epsilon\)

Table 2—First-Stage Least-Squares Regressions of Earnings on the Instrumental Variables
taken as a whole and here we can derive at least some comfort from the relative stability of the relations over time. There is evidence of drift over time in some of the coefficients; but the key coefficients with one exception (the debt coefficient of 1957) are of the same sign, order of magnitude, and approximate level of significance in all three years.\textsuperscript{38}

D. The Second-Stage Estimates

Table 3 presents the second-stage estimates with the earnings variable constructed from the full set of instrumental variables using the regression coefficients in panel II of Table 2. In the upper half of Table 3 the regressions have been fitted with a constant term so as to provide a check on the validity of our specification that the true constant term is zero. Comparison with the corresponding panel of Table 1 shows that the constant term does indeed drop sharply in the two years, 1957 and 1954, for which it was substantial in the direct least-squares regressions. One coefficient changes sign and all three constants are now much smaller than their standard errors.\textsuperscript{39} We have therefore considerable support for our hypothesis that the true constant term is zero; and that the large positive constants of the direct least-squares regressions are simply a reflection of the proxying power of total assets when earnings are measured with error. Further confirmation of the errors model (as well as of our conjecture that the errors of measurement in earnings would not be strongly correlated with our instrumental variables) is

\textsuperscript{38} Although the coefficients in the second set of regressions in Table 2 are relatively stable over time, considerable difference exists between the two sets in the values of the coefficients, particularly of debt and preferred stock. There is, however, nothing anomalous or surprising in these differences. With dividends excluded, the debt and preferred coefficients in the first set show the effects on total earnings of differences in debt and preferred \textit{given total assets}, i.e., essentially the effects of differences in "capital structure." With dividends included, and the constant term close to zero, the debt and preferred coefficients in the second set are largely showing the effects of differences in debt and preferred \textit{given net profits}, i.e., essentially the proxying power of debt and preferred for their corresponding pieces of the income stream.

\textsuperscript{39} Since no "canned" two-stage least-squares computer program was available to us at the time the estimates were made, the coefficients appearing in Table 3 (and in all subsequent second-stage tables) were computed literally in two stages. That is, a new earnings variable was actually constructed from the first-stage regressions and the second-stage regression run with that variable included among the independent variables. The coefficients so obtained are, of course, exactly the same as those that would have been obtained from a 2SLS program, but that is not true for the standard errors of the coefficients (or the standard error of the regression). In terms of previous notation, the sampling variances in the literal second stage will be the product of the inverse of the moment matrix and the scalar \textit{Var}(\textit{u}+\alpha\bar{e}); whereas the appropriate scalar multiple for our specification should be \textit{Var}(\textit{u}−\alpha\bar{e}). An estimate of the latter is provided by the residual variance of the direct least-squares equation recomputed with the 2SLS coefficients. In those cases in which the constant term in the regression is suppressed the desired residual variance must, of course, be computed around the presumed mean of zero under the specification, rather than around the actual sample mean residual.

We are grateful to Albert Madansky for some helpful discussion on these and certain related points.
### Table 3—Second-Stage Estimates with Computed Earnings

**Dependent Variable: \((V - \tau D) / A\)**

<table>
<thead>
<tr>
<th>Year</th>
<th>Coefficients of</th>
<th>Mult. R</th>
<th>Adjusted Standard Error</th>
<th>Ratio of Adjusted Standard Error to Mean (V/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Computed Earnings&lt;sup&gt;b&lt;/sup&gt; ((X^r - \tau R) / A)</td>
<td>Size (1/A \cdot 10^n)</td>
<td>Growth (\Delta A / A)</td>
</tr>
<tr>
<td>1957</td>
<td>- .004 (.08)</td>
<td>16.2 (1.7)</td>
<td>- .280 (.08)</td>
<td>1.37 (.24)</td>
</tr>
<tr>
<td>1956</td>
<td>.054 (.08)</td>
<td>15.6 (1.6)</td>
<td>- .122 (.07)</td>
<td>.847 (.23)</td>
</tr>
<tr>
<td>1954</td>
<td>.072 (.10)</td>
<td>18.1 (2.2)</td>
<td>- .234 (.07)</td>
<td>.286 (.17)</td>
</tr>
</tbody>
</table>

* Constrained to equal zero.

<sup>b</sup> From Panel II of Table 2.
provided by the behavior of the earnings coefficients, all of which are higher than their counterparts in Table 1—particularly so in 1954 where the coefficient increases by nearly 40 per cent.

The second part of Table 3 shows the second-stage regressions with constant term constrained to equal zero. Since the a priori value of the constant term in our model is zero and since the actual sample values, as we have seen, do not differ significantly from zero, these constrained estimates are the most efficient obtainable under our specification. Notice that the loss of efficiency from the use of instrumental variables turns out to be relatively small. In the second-stage regressions, the standard errors of the coefficients average only about 3 per cent higher than those of the direct least-squares regressions. As it turns out, the earnings coefficients also show only a very slight increase over those in the corresponding part of Table 1. This is, of course, not entirely surprising. Since the bias due to measurement error takes the form of an attenuation of the coefficient of the “noisy” earnings variable, the mere act of suppressing the constant will, as noted earlier, automatically force that slope much closer to its “true” value. Only where the error of measurement is large, as it appears to be in 1954, or where one or more of the other explanatory variables in the equation are strongly correlated with the unobservable \( X^* \) are the effects of removing the bias likely to stand out sharply. In our present specification it happens that the other variables are not good proxies for earnings; but, as we saw in the case of the constant term and as we shall see even more strikingly in the next section, such proxies and consequent dangers of bias do enter as soon as we try to test the validity of that specification.

E. Some Further Tests of the Specification

1. Debt and valuation. It will be recalled (see Sec. I. C) that under our model the value of the firm, growth aside, is equal to capitalized tax-adjusted earnings plus \( \tau \) times the market value of the debt. Since the coefficient of the debt variable is thus specified to have the value \( \tau \) independent of the other coefficients, we have so constrained it in all the regressions by subtracting \( \tau D \) from \( V \) in forming the dependent variable. For most finance specialists, however, this will be regarded as a gross misspecification. The traditional view in finance has long been that

\[ \text{40} \] It may perhaps be worth mentioning that in terms of their ability to explain \( V \)—one measure of this ability being indicated by the relative standard errors shown in the last column of Table 3—the second-stage results here presented compare favorably with those obtained in other valuation studies such as, e.g., Gordon [7].

\[ \text{41} \] As a further check on the earnings coefficient, and in particular to see whether our use of dividends as an instrumental variable might be introducing some distortions, we ran the second-stage regressions with \( \bar{X} \) constructed from the first panel of Table 2. The earnings coefficients turned out to be almost identical to those in Table 3, but, of course, the standard errors were considerably higher. We also have run the entire set in undeflated form and again found no significant change in the coefficients.
because of the supposed nontax advantages of leverage the contribution of debt to value, given tax-adjusted earnings, will be significantly greater than \( r \); and that there will also be a substantial positive leverage contribution from preferred stock (whose coefficient, of course, we have in effect constrained to be zero by omitting any preferred stock variable in the value equations). A complete test of these conflicting views about the effects of leverage on valuation would be far too space-consuming for this paper. We shall, therefore (albeit somewhat regretfully), have to postpone any full-scale attempt to settle the leverage issue to a subsequent paper. Fortunately, however, there is no need for present purposes to have the most precise estimate possible of the leverage effect. At the moment, the concern is merely whether our specification is a sufficiently accurate first approximation for purposes of estimating the cost of capital; and this more narrow question can be answered by the relatively simple expedient of including debt and preferred stock among the independent variables and seeing whether they make any significant additional contribution.

The results of this test are presented in Table 4. The top panel shows the direct least-squares estimates using measured earnings. Clearly, if these results were to be taken seriously, our specification would have to be rejected. In one of the three sample years, 1954, the preferred stock coefficient is positive and one and one-half times its standard error, while the debt coefficient is positive and slightly greater than three times its standard error. Taking the three years as a whole, the average debt coefficient is about .13 and the average preferred stock coefficient about .07—values uncomfortably far from zero in the present context. Note also that including these capital structure variables completely reverses the time pattern of the key earnings coefficients with the values increasing steadily from 1954 instead of decreasing as in the second panel of Table 1 or as in Table 3.

Previous results and analysis, however, would lead us to suspect upward bias in these coefficients, particularly in 1954. If the measurement error in 1954 earnings is really as serious as it seemed to be in earlier tests, the proxying power of debt and preferred stock for their corresponding components of the true earnings stream could easily produce positive values as large or larger than the coefficients actually observed in the DLS equation. A glance at the two-stage results in the second panel of Table 4 strongly suggests that such proxying is very definitely at work in the DLS equations. Notice that all six coefficients fall in size—the fall being particularly dramatic in the case of the large debt coefficient of 1954. The average of the three debt coefficients is now only .01 and the average of the preferred stock coefficients a little over .025. The average standard error is many times as large and the coefficients themselves vary unsystematically in sign and magnitude over the
<table>
<thead>
<tr>
<th>Year</th>
<th>Earnings ($Y - \bar{Y}$)/$A$</th>
<th>Size ($\bar{X}/A$)</th>
<th>Growth $\Delta X/A$</th>
<th>Debt $D/A$</th>
<th>Preferred Stock $P/A$</th>
<th>Multi. $R^2$</th>
<th>Adjusted Standard Error</th>
<th>Ratio of Adjusted Standard Error to Mean $Y/A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>16.3 (.92)</td>
<td>$-2.774 (.08)$</td>
<td>$-0.103 (.15)$</td>
<td>$0.02 (.15)$</td>
<td>$0.05 (.15)$</td>
<td>.88</td>
<td>.057</td>
<td>0.052</td>
</tr>
<tr>
<td>1956</td>
<td>15.8 (.94)</td>
<td>$-1.33 (.08)$</td>
<td>$0.00 (.14)$</td>
<td>$0.06 (.14)$</td>
<td>$0.07 (.14)$</td>
<td>.88</td>
<td>.057</td>
<td>0.051</td>
</tr>
<tr>
<td>1954</td>
<td>15.4 (.12)</td>
<td>$-2.12 (.09)$</td>
<td>$1.31 (.12)$</td>
<td>$1.86 (.12)$</td>
<td>$1.86 (.12)$</td>
<td>.80</td>
<td>.060</td>
<td>0.050</td>
</tr>
</tbody>
</table>

* Constant constrained to equal zero.

b Earnings constructed from Panel II of Table 2.
three years. We clearly, therefore, have considerable support for the proposition that we are seeing here only sampling fluctuations around a true value of zero; and that the coefficients of these variables may safely be constrained to zero in the interests of increasing the efficiency of the estimate of the key earnings coefficient.\footnote{It should be noted, in this connection, that the debt and preferred stock coefficients are likely to be somewhat biased upward even in the second-stage regressions. The bias arises from the fact that we now have the debt and preferred variables on both sides of the equation so that any errors of measurement or of concept will be positively correlated.}

2. Dividends and valuation. The treatment of dividends so far in this paper has been predicated on the assumption that the true, as opposed to the purely informational, effect of dividends on market value is small enough in this industry to be safely ignored. By safely ignored we mean both that it would not be worth the bother to introduce payout terms in the expression for the cost of capital; and that there is no significant loss in the efficiency of estimation of the key earnings coefficient by omitting dividends entirely from the value equation. The two-stage, instrumental variable approach permits us to test these suppositions and in the process to shed some new light on the long-standing controversies in the field of finance over the role of dividends in valuation.

The dividend variable to be used in the tests will be of the form

\[
\left( \frac{\text{div}}{A} - \lambda \left( \frac{\overline{X} - \tau R}{A} \right) \right),
\]

where \(\lambda\) is the sample average "payout" ratio in terms of \((\overline{X} - \tau R)\) for the year in question. This payout form for the dividend variable has the advantage of preserving the interpretation of the earnings coefficient as the capitalization rate for companies following an average dividend policy and thereby facilitating comparisons with previous results.\footnote{If \(\text{div}/A\) is used as the dividend variable, the earnings coefficient measures instead the effect of an increase in earnings, dividends held constant (i.e., of an increase in earnings all of which is retained). The coefficients of the two regressions

\[
V - \tau D = a_1 \left( \frac{\overline{X} - \tau R}{A} \right) + a_2 \left( \frac{\text{div}}{A} \right) + \ldots
\]

and

\[
V - \tau D = a_1' \left( \frac{\overline{X} - \tau R}{A} \right) + a_2' \left( \frac{\text{div}}{A} \right) - \lambda \left( \frac{\overline{X} - \tau R}{A} \right) + \ldots
\]

are, of course, not independent and in fact it can easily be verified that \(a_2' = a_2\), \(a_1' = a_1 + \lambda a_2\), \(\text{Var}(a_2') = \text{Var}(a_2)\), and \(\text{Var}(a_1') = \text{Var}(a_1) + \lambda^2 \text{Var}(a_2) + 2 \lambda \text{ Cov}(a_1, a_2)\).

Note also that the optimal value of \(\lambda\) to use would be the regression coefficient of dividends on earnings with all the other explanatory variables included in the regression. Since a constant}
The results of the tests are presented in Table 5. The top panel shows the direct least-squares results with measured earnings as the earnings variable. Notice that the dividend coefficient is positive in all three years and very substantially so relative to its standard error and to the earnings coefficient in 1954. Findings of this kind (which are quite typical of past valuation studies) have been the main empirical support of the traditional view that high dividend payouts increase the value of the firm.\footnote{Since the conventional tests in the literature involving the regression of some value variable (usually share prices) on earnings and dividends do not suppress the constant term, we reran all the DLS equations in Table 5 with constant included to see whether that would make any significant difference. It didn't.} If our analysis is correct, however, these coefficients are biased upwards because of the additional information that dividends convey about expected future earnings beyond that contained in the imperfectly measured earnings variable.

The second-stage results in the bottom panel of Table 5 provide a test of this interpretation. Since dividends are included among the instrumental variables, the coefficient of the dividend variable in the second stage should provide an estimate of the contribution of dividends to valuation that is presumably less seriously contaminated by the mere informational effect. In this light the results are quite striking. In each of the three years the point estimate of the coefficient is now found to be negative, although of small magnitude and significance, except possibly in 1954. Thus, at least for this industry in these years the traditional view that the market valuation of shares tends to increase with the proportion of income paid out as dividends is completely unsupported.\footnote{We can see, however, some possible mechanisms that might lead to a negative bias for the dividend coefficients under the two-stage approach. Consider, for example, the case of a company whose payout ratio is substantially greater than the average for the industry as a whole. Then since dividends enter the first-stage regression with substantial positive weight, the computed value of $X$ for that company will tend to be pushed above its measured $X$ and probably also its true $X^*$\footnote{In an effort to determine how important such a bias might be we ran two kinds of supplementary tests. One was a simulation in which the true dividend effect was set at zero and in which about 10 per cent of the firms in the sample were assumed to have payout ratios widely divergent from the industry mean. The result was a very slight negative coefficient, but very far from being statistically significant. The second test consisted of extending the two-stage approach to the dividend variable itself and was motivated, of course, by the fact that the bias described above is closely akin to that occasioned by ignoring the fact that dividends are really an endogenous variable from the standpoint of the system as a whole (cf. n. 31).}. Hence, in the second stage the value of $V$ predicted from $X$ will tend to be too high; and since this positive deviation will be associated with above-average dividends, the dividend coefficient will tend to be pushed down so as to reduce the size of the over-all residual. There will, of course, be a similar negative twist imparted to the dividend coefficient in the opposite case of payouts much below the average.}
<table>
<thead>
<tr>
<th>Year</th>
<th>Earnings $(\bar{Y} - rD)/A$</th>
<th>Coefficients of</th>
<th>Growth $\Delta A/A$</th>
<th>Dividend Policy $(\bar{Y} - \bar{D})/A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>16.0 $(.46)$</td>
<td>$R$ $.88$</td>
<td>$\bar{A}$ 1.41 $(.24)$</td>
<td>$\Delta A$ -9.15 $(2.5)$</td>
</tr>
<tr>
<td>1956</td>
<td>16.6 $(.40)$</td>
<td>$(2.3)$</td>
<td>$\bar{A}$ 0.96 $(.22)$</td>
<td>$\Delta A$ -6.49 $(2.3)$</td>
</tr>
<tr>
<td>1954</td>
<td>18.9 $(.42)$</td>
<td>$(2.1)$</td>
<td>$\bar{A}$ 0.67 $(.17)$</td>
<td>$\Delta A$ -6.37 $(2.1)$</td>
</tr>
</tbody>
</table>

*Constant constrained to equal zero.

b Earnings constructed from Panel II of Table 2.
For present purposes of estimating the cost of capital, however, perhaps the main conclusion to be drawn from Table 5 is that the dividend effect is sufficiently small and uncertain to be safely neglected. From the statistical point of view the addition of this variable does not appear to improve the reliability of the estimates of the other coefficients, notably that of the critical earnings variable. Indeed, even the small reduction in the residual variability resulting from adding the dividend variable is more or less offset by the loss of one degree of freedom, so that on balance the standard errors of the regression coefficients actually tend to be somewhat higher. And from the substantive point of view, even if one were to accept the coefficients of the dividend variable as a reliable measure of the true dividend effect—which we would certainly hesitate to do in view of the large standard errors and the puzzling drift of the coefficients over time—\textsuperscript{47} the implied quantitative effect of dividend policy on market valuation and the cost of capital would still be negligible. In 1954, for example, the year with the largest dividend coefficient in absolute value, the elasticity of the value of the shares at the mean with respect to dividends paid is only about $-0.08$. That is, a 20 per cent decrease in dividend payout—which would constitute a very large decrease in this industry, since the standard deviation of the payout ratio is about 10 per cent of the mean—would be associated with an increase in the value of the shares of only about one and one-half per cent, an amount far smaller than the relative standard error of estimate of $S$ itself.\textsuperscript{48} Accordingly, in the discussions of valuation and the cost of capital to follow we shall work exclusively with the results in Table 3 rather than those in Table 5.

Operationally, this meant regressing both $X$ and dividends on the instrumental variables in the top panel of Table 2 and then regressing $V$ on computed $X$ and computed dividends (along with size and growth) in the second stage. The result turned out to be that in all three years the dividend coefficient was less negative than those of Table 5, and in 1957 the coefficient was actually positive and of essentially the same order of magnitude as the direct least-squares estimate. Although these results tend to confirm our suspicions about possible bias of the results in Table 5, we feel they also show that the bias is unlikely to be large enough to force us to revise our conclusion, namely, that the data rule out any substantial positive effect for dividends along the lines of the traditional literature in finance [10, esp. p. 363] or of Gordon [6] or Durand [5]. In this connection, we find it encouraging to note that other researchers are now also beginning to report that the dividend effect is actually relatively small even in the utility industry (see, e.g., Friend and Puckett in [5a] and Beranek [1a]).

\textsuperscript{47} The only plausible rationalization for the drift that we have been able to find is that it may be reflecting the fact that the number of companies paying tax-exempt dividends (cf. n. 19) increased steadily after 1954.

\textsuperscript{48} It may seem strange to some that we test for the effect of dividends on share prices in an equation describing total market value (including debt and preferred stocks) rather than one solely in terms of the value of the common shares. Remember, however, that under our valuation model, the variables debt and preferred stock must be included among the explanatory variables in the $S$ relation (cf. above Sec. I.C). Since these variables are also among the
3. Comparison with estimates from the yield formulation. As noted earlier, an alternative way of estimating the cost of capital under our specification would be to use a "yield" equation of the form

\[
\frac{\bar{X}_t - \bar{R}}{V - \tau D} = a'_1 + a'_2 \frac{1}{V - \tau D} + a'_3 \frac{\Delta \bar{A}}{V - \tau D},
\]

with the constant term \(a'_1\) providing the desired estimate of \(\rho\) (Cf. Sec. II.A and n. 28.) The main drawback to this approach comes from the presence of \(V - \tau D\) in the denominators of variables on both sides of the equation which imparts an upward bias to the coefficients of the independent variables and a consequent downward bias to the crucial constant term. Since the direction of the bias is known, however, we can use equations of this form to provide at least a rough check on the reasonableness of the estimates obtained by the more roundabout, two-stage approach.

The coefficients obtained in the yield equations are presented in Table 6. To facilitate comparison with the estimates in Table 3 a column has been added showing the reciprocal of the constant term, which is the estimate of the capitalization factor for earnings implied by the observed constant terms in the yield equations.\(^4\) As predicted under our model, the capitalization factors obtained via the yield equations are indeed all higher than those obtained via the two-stage approach. The gap between the two sets of estimates tends to widen somewhat over time, but the differences are never very large. This close agreement should remove any lingering fears that major distortions in the estimates may somehow have been introduced in the two-stage approach.\(^5\) At the same time, it suggests that the simpler yield equations may still have

instrumental variables, a second-stage S equation containing computed profits, debt, preferred and dividends would be just identified and that only by virtue of the constant term included in the first stage but suppressed in the second. Since, as we have seen, the contribution of total assets as an instrumental variable is very small when dividends are included (along with debt and preferred) computed profits will be so nearly a linear combination of the other explanatory variables that no effective identification is possible. This extreme collinearity is avoided in the \(V\) tests, of course, since debt and preferred are excluded from the \(V\) equation along with the constant term. There is some possibility of success in designing dividend tests in terms of \(S\) by the use of more elaborate kinds of instrumental variables; but further discussion of these is best postponed to sequel papers. Note also that the practical necessity of excluding at least two of the instrumental variables in order to obtain adequate identification explains why we have not been able to obtain any joint tests of the various specification restrictions considered above singly.

\(^4\) It can be shown (see Cramér [4, pp. 353–54]), that given a sample estimate \(\hat{a}\) with standard error \(\sigma\), the corresponding (large sample) estimate of \(1/\sigma\) is \(1/\hat{a}\) and of its standard error, \(\sigma/\hat{a}^2\).

\(^5\) Initially we thought it might be possible to provide further tests of the specification by exploiting the information in the residuals of the estimating equations. For each sample
Table 6—Direct Least-Squares Estimates from Yield-Form Equations

Dependent Variable: \((X^r - rR)/V - rD\)

<table>
<thead>
<tr>
<th>Year</th>
<th>Coefficients of</th>
<th>Mult. R</th>
<th>Reciprocal of Constant Term and Its Implied Standard Error</th>
<th>Ratio of Standard Error of Regression to Mean of Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size (1/V - rD) (.10^6)</td>
<td>Growth (\Delta A/V - rD)</td>
<td>(\Delta R)</td>
<td>(\Delta R)</td>
</tr>
<tr>
<td>1957</td>
<td>.0592 ((.002))</td>
<td>.166 ((.04))</td>
<td>.0516 ((.02))</td>
<td>.58</td>
</tr>
<tr>
<td>1956</td>
<td>.0582 ((.001))</td>
<td>.066 ((.04))</td>
<td>.0325 ((.01))</td>
<td>.39</td>
</tr>
<tr>
<td>1954</td>
<td>.0506 ((.001))</td>
<td>.121 ((.04))</td>
<td>.0124 ((.01))</td>
<td>.45</td>
</tr>
</tbody>
</table>

A useful role to play in valuation studies, particularly where the interest is mainly in determining the direction of changes in the cost of capital over time rather than developing precise estimates or testing the basic specification as developed here.

F. Summary

In this section we have attempted to apply and to test the model of valuation developed in Sections I and II of this paper. Strong support has been presented for our contention that the usual direct least-squares value equations (and yield-form equations) lead to biased estimates of the key parameters. The evidence has also been seen to be consistent with our specifications with respect to the constant terms, the value of the debt coefficient and the role of dividends. Given these specifications, the most efficient, consistent estimates of the value relation are those shown in the second panel of Table 3; and it is to the economic interpretation of these results that we now turn.

Year we have three residuals which can be expressed in previous notation as \(u_{t} = u_{t} - \alpha \sigma_{t}, u_{t} = u_{t} + \phi_{t},\) and \(u_{t} = u_{t} + \gamma_{t}.\) If, therefore, we could assume \(E(u_{t} \sigma_{t}) = E(u_{t} \phi_{t}) = E(u_{t} \gamma_{t}) = 0\) for \(t, \tau = 1957, 1956,\) and \(1954,\) we could obtain estimates of the variances of the specification and measurement errors for each year as well as of the interyear correlations of the specification and measurement errors. As it turns out, however, the assumption that the side correlations are zero is clearly untenable (there being particularly strong evidence of substantial negative correlation between \(v\) and \(w\)).

Although the residuals thus cannot be exploited to get directly at the specification error, the question arises as to whether they can at least be exploited to improve the efficiency of the estimates of the coefficients along the lines suggested in Zellner and Theil [23], Zellner [22], or Telser [20]. Even if there were no conceptual difficulties in applying these methods to models involving measurement error it seems very doubtful that the gain in efficiency would be worth the cost in our case because of the very high year-to-year correlation between the independent variables.
IV. Valuation and the Cost of Capital in the Utility Industry

A. The Anatomy of Valuation

In Table 7 the coefficients from Table 3 have been multiplied by the sample mean values of the variables so as to illustrate the relative importance of the various factors contributing to the market value of the typical firm in the industry. As expected, by far the largest component of market value is the capitalized earning power of the assets currently held. Next in importance is the tax subsidy to debt reflecting, of course, both the high corporate tax rate during the sample years and the very high debt ratios characteristic of this industry. Future growth potential, though small, was apparently increasing steadily in importance relative to current earning power over this period and by 1957 was accounting for something over 10 per cent of market value. The only mild surprise perhaps is the virtually negligible contribution of size to value. The difference in valuation, other things equal, between a firm of indefinitely large size and that of mean size in our sample turns out to be only on the order of 1 or 2 per cent of total market value. It should be remembered, however, that by ordinary standards, all the firms in our sample are extremely large.\(^51\)

B. The Cost of Equity Capital

Turning now from valuation to the other side of the coin, the cost of capital, we show in Table 8 the estimates of the cost of equity capital implied by the earnings coefficients of Table 3.\(^62\) For comparison, the table also shows two other measures of the cost or “ease of acquisition”

\(^{51}\) To see whether a more direct size variable than our \(1/A\) might lead to different results we reran the equations using \(\log A\) as the measure of size (in the deflated equations). No significant change occurred either in the relative importance of the size effect or in the values of other coefficients.

\(^{62}\) Cf. note 49.
TABLE 8—Estimated Cost of Equity Capital and Some Alternative Measures of Equity Costs

<table>
<thead>
<tr>
<th>Year</th>
<th>Estimated Cost of Equity Capital ($\rho$)</th>
<th>Average Earnings Yield on Shares ($\bar{\pi}/S$)</th>
<th>Reciprocal of Price to Book-Value Ratio ($B/S$)</th>
<th>Average Tax- and Leverage-Adjusted Total Earnings Yield ($\bar{X}^r - \tau \bar{R})/(V - \tau D$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
</tr>
<tr>
<td>1957</td>
<td>.062</td>
<td>122</td>
<td>.070</td>
<td>106</td>
</tr>
<tr>
<td>1956</td>
<td>.060</td>
<td>118</td>
<td>.070</td>
<td>106</td>
</tr>
<tr>
<td>1954</td>
<td>.051</td>
<td>100</td>
<td>.066</td>
<td>100</td>
</tr>
</tbody>
</table>

of equity capital frequently used by economists in investment studies, namely, the average earnings-price ratio and the reciprocal of the average price-book value ratio for the shares. Notice that all three measures indicate a rise in the cost of equity capital between 1954 and 1957, but our measure indicates a steeper and more substantial increase over the interval. The causes and implications of this apparently lesser responsiveness of the standard measures will become clear in subsequent discussion.

Insofar as levels are concerned, notice that the average earnings yield happens to be consistently higher than our estimate of the cost of equity capital. We say "happens to be" to emphasize that under our model of valuation there is no "normal" or even simple relation to be expected between the two concepts. The earnings yield for any company is not a given fixed number for each member of the class, as so much of the traditional discussion in finance and economics would suggest, but rather a function whose arguments include the cost of equity capital for the class, the firm’s growth potential, its leverage policy, and its size. The sample mean earnings yield shows only the combined effect of these different and to some extent offsetting influences.

The precise way in which the various components of the yield fit together in the present sample is illustrated in Table 9. The entries have been computed by "solving" our fitted $V$ equations for their implied $\bar{\pi}/S$ functions, and then substituting the appropriate sample mean values for the variables into these derived functions. Specifically, if the $V$ equation (neglecting the deflator) is represented as

$$V - \tau D = -a_0 + a_1(\bar{X}^r - \tau \bar{R}) + a_2 \Delta A;$$

then, making use of the identities $V = S + D + P$ and $\bar{X}^r = \bar{\pi} + P\sigma + \bar{R}$, we have as the implied function for the value of the shares (cf. II. C):
\[(20) \quad S = -a_0 + a_1 \bar{\pi}^r - (1 - \tau) [D - a_1 \bar{R}] - [P - a_1 \bar{P}d\bar{v}] + a_2 \Delta A, \]

and as the implied earnings yield function:

\[(21) \quad \frac{\bar{\pi}^r}{S} = \frac{1}{a_1} - \frac{\left(\frac{a_2}{a_1}\right) \Delta A}{S} + \frac{(1 - \tau) \left(\frac{D}{a_1} - \bar{R}\right)}{S} \]

\[+ \frac{\left(\frac{P}{a_1} - \bar{P}d\bar{v}\right)}{S} + \frac{\left(\frac{a_0}{a_1}\right)}{S}. \]

The first term of (21) is the cost of equity capital in our sense for the class as a whole; the second is the growth effect; the third and fourth, the leverage effects for debt and preferred stock, respectively; and the last term is the size effect. The presence of leverage, by raising the "risk" attaching to the shares, drives up the expected earnings yield relative to the cost of equity capital. The existence of growth potential with its implied prospect of future capital gains tends to reduce the current expected earnings yield on the shares.

As can be seen from Table 9, the leverage effect was the dominant one over the period as a whole which, of course, explains why the earnings yield consistently overstated the cost of equity capital during this period. The substantial narrowing of the gap between 1954 and 1957 reflects the large rise in the growth contribution after 1954 both in absolute and relative terms.

C. Valuation, Growth, and the Cost of Equity Capital

Although the conventional earnings/price ratio is thus distorted as a measure of the cost of equity capital both by the growth effect and the

<table>
<thead>
<tr>
<th>Table 9—Relation Between Average Earnings Yield and the Cost of Equity Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Earnings Yield</td>
</tr>
<tr>
<td>Less:</td>
</tr>
<tr>
<td>Leverage Effect</td>
</tr>
<tr>
<td>Debt</td>
</tr>
<tr>
<td>Preferred Stock</td>
</tr>
<tr>
<td>Size Effect</td>
</tr>
<tr>
<td>Plus:</td>
</tr>
<tr>
<td>Growth Effect</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>Estimated Cost of Equity Capital</td>
</tr>
</tbody>
</table>
leverage effect, it is important to remember that the real culprit is the growth effect. That is, it is the growth effect and not the leverage effect that keeps us from being able to infer the cost of equity capital directly from market yields. We know, for example, that if there were no growth and no taxes the fundamental valuation equation, expressed in yield form, would be simply $X/V = \rho$. The ratio of expected total earnings to total market value—which may be thought of as a "leverage-corrected" yield—would thus provide a direct estimate of $\rho$. In principle, any firm could so approximate its cost of equity capital from its own company data, although, of course, as a practical matter, a better estimate would be obtained by averaging over a large group of similar firms so as to wash out any random noise in $X$ or $V$. When we allow for taxes and the consequent tax subsidy on debt, the picture becomes slightly more complicated, but a direct approximation of $\rho$ still exists. The appropriate yield [cf. (6) above] now becomes $(X' - \tau R)/(V - \tau D)$—the ratio of total tax-adjusted earnings to total market value minus the value of the tax subsidy.

This method of direct approximation breaks down, however, in the presence of growth. The leverage-adjusted yield will be systematically too low as an estimate of $\rho$ for any company with growth potential, as will be the group average yield for any sample that contains significant numbers of growth companies. Nor will the movements of the yield over time conform well with changes in $\rho$ to the extent that the market's evaluation of future growth potential changes over time (and, of course, much of the short-term variation we see in share prices stems precisely from this source). Some idea of how sizable the distortions of level and movement of the yield relative to $\rho$ can be—even in such a low-growth industry as our electric utilities and even over such a short span of time—can be gained by comparing our estimates of the cost of equity capital in the first column of Table 8 with those of the tax- and leverage-adjusted yield, $[(X' - \tau R)/(V - \tau D)]$, in the last column of that table.

One somewhat surprising aspect of this comparison that is perhaps worth some special mention is the relative stability of the yield series over this period. Because of the many uncertainties surrounding estimates of future growth potential and because of the sensitivity of current market values to even small changes in projected growth rates one would expect the growth component in the denominator of the yield ratio to be quite volatile; and hence that the market yield would tend to swing quite substantially in response to these continuing re-evaluations. Some idea of why this "normal" pattern did not obtain during our sample period can be gained from Figure 1. The solid line functions there plotted are the basic value regressions of Table 3 for the beginning and ending years, 1954 and 1957, expressed in ratio form as
\[
\frac{V - \tau D}{\bar{X}^r - \bar{\tau R}} = a_1 + a_2 \frac{\Delta A}{\bar{X}^r - \bar{\tau R}}
\]

(and hence ignoring the minor size effect). The dependent variable is thus the reciprocal of the tax- and leverage-adjusted yield; the intercept \(a_1\) is our estimate of \(1/\rho\); and the slope \(a_2\) is the coefficient of growth in the \(V\) equation.\(^5\)

Notice that in 1954, at the beginning of the period, the market's estimate of the growth potential of the industry was quite low. Because the slope was so flat, the approximate sample mean value of \((V - \tau D)/(\bar{X}^r - \bar{\tau R})\)—indicated by the circled cross—differed only very slightly from the estimate of \(1/\rho\) implied by the intercept. By 1957, however, a striking increase had taken place in the market's valuation of future

\(^5\) The use of the reciprocal of the yield rather than the yield itself is simply a matter of convenience since the presence of growth impounded in \(V\) would lead to a nonlinear relation between \((\bar{X}^r - \bar{\tau R})/(V - \tau D)\) and growth as measured by \(\Delta A/(V - \tau D)\).
growth in the industry. As can be seen from the broken line—which has been plotted with 1954 intercept and 1957 slope—this large revaluation would have pushed the average value of \((V-\tau D)/(\bar{X}r-\tau \bar{R})\) up by nearly 15 per cent to 22.4 (equivalent to a yield of about .044) if no other changes had occurred. But instead of this “pivoting” around a stable intercept of \(1/\rho_{st}\), our estimates indicate that there happened to take place a simultaneous and quite substantial drop in the intercept (i.e., rise in the cost of equity capital). So substantial, in fact (when combined with the slight fall in the mean value of the growth variable itself), that the upward push of the revaluation of growth was more than offset; and the mean value of \((V-\tau D)/(\bar{X}r-\tau \bar{R})\) actually fell by about 10 per cent.

Although these compensating movements in \(\rho\) and the market’s evaluation of growth “explain” the relative stability of the tax- and leverage-adjusted yield during the sample period, the explanation may strike the reader as having a somewhat paradoxical flavor. Growth potential, after all, is the opportunity to invest in the future in projects whose rates of return exceed the cost of capital. One would expect, therefore, that a rise in the cost of capital would normally be associated with a fall in growth potential. There are a number of possible explanations for the opposite behavior in the present instance, but discussion of them is perhaps best postponed until we have first provided estimates of the average cost of capital relevant for investment decisions.

D. The Required Yield or Average Cost of Capital

As emphasized earlier, the relevant cost of capital for investment decisions at the level of the firm is the average cost of capital, \(\rho(1-\tau L)\), where \(L\) measures the “target” proportion of debt in future financing. The average cost is thus not a fixed number, but a schedule or function whose arguments are \(\rho\) (which is an “external” property of the class or industry determined by the market) and \(L\) (which is a matter of “internal” company policy). Although the average cost of capital, unlike the cost of equity capital, is thus in principle different for each firm in the industry, we can get some idea of its value and behavior for the typical electric utility by using a typical or average value for \(L\). The obvious candidate, of course, is the actual sample average of \(D/A\) for each year, since \(D/A\) measures the average proportion of debt in past financing, and this proportion is likely to be quite stable (particularly when averaged over the industry). Estimates with these values for \(L\) are shown in block 1 of Table 10. (N.B. For notational convenience we shall hereafter refer to these estimates as \(C(D/A)\), using \(C(L)\) to mean the function itself, and \(C(D/A)\), i.e., without the bar on \(D/A\), to refer to
Table 10—The Average Cost of Capital and Some Comparison Series

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Cost of Capital $C(D/A)$</th>
<th>Yields on AAA Public Utility Bonds*</th>
<th>Sample Mean Yield on Debt $(R/D)$</th>
<th>Sample Mean Yield on Preferred Stock $(Pd/P)$</th>
<th>Weighted Average Yields With Book-Value Weights $C_B(D/A)$</th>
<th>Weighted Average Yields With Market-Value Weights $C_M(D/A)$</th>
<th>Growth Adjusted Average Yield $\left(\bar{X} - \gamma \bar{R}\right)/\sqrt{V - G}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
<td>Amt. As per cent of 1954</td>
</tr>
<tr>
<td>1957</td>
<td>.046 128</td>
<td>.043 137</td>
<td>.029 112</td>
<td>.047 115</td>
<td>.034 104</td>
<td>.042 108</td>
<td>.047 122</td>
</tr>
<tr>
<td>1956</td>
<td>.045 125</td>
<td>.039 126</td>
<td>.029 112</td>
<td>.046 112</td>
<td>.035 106</td>
<td>.043 110</td>
<td>.045 115</td>
</tr>
<tr>
<td>1954</td>
<td>.036 100</td>
<td>.031 100</td>
<td>.026 100</td>
<td>.041 100</td>
<td>.033 100</td>
<td>.039 100</td>
<td>.039 100</td>
</tr>
</tbody>
</table>


b With $G$ equal to growth coefficient in Table 3, Panel 2, times sample mean value of $\Delta A/A$. 

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the function evaluated not at the industry mean but at a particular company's value of $D/A$.)

These estimates $C(D/A)$ of the average cost of capital are, of course, always below the corresponding estimates of $\rho$ (column 1 of Table 8); but the movements over time of the two series are closely similar since, as expected, the sample mean value of $D/A$ is quite stable.\footnote{In interpreting these estimates of the average cost of capital it may be worthwhile to remind readers that the figures in column 1 represent tax-adjusted yields (cf. notes 8 and 11) and are not adjusted for size (i.e., they would apply, strictly speaking, only to firms of infinite size). Adjusting to the average size of firm in our sample and converting to the more familiar before-tax basis, the corresponding average costs would be .073, .087 and .090, respectively, for 1954, 1956, and 1957.} Notice also that the estimates of $\rho$ and hence of $C(D/A)$ conform quite closely in their movements with the average yield on AAA bonds in the industry (block 2 of Table 10)—probably the most popular surrogate for the cost of capital in investment studies. This conformity is particularly interesting since the rate of interest on bonds enters only very indirectly into our calculations of $\rho$ and $C(D/A)$ and, as can be seen from block 3, the implied average rate of interest in our sample does not even seem to conform well with the AAA series.\footnote{One important reason for the divergence between market rates on bonds and our implied average yields is the fact that our debt variable in the denominator contains substantial and varying amounts of short-term liabilities.} From the economic point of view, this parallelism between movements in $\rho$ and the AAA yields would seem to suggest that over this short interval at least the movements of both series were dominated by factors affecting the supply of and demand for capital generally. Changes, if any, in investors’ tastes for risk-bearing or in their evaluation of the riskiness of this industry in relation to others were apparently not large enough to cause any significant divergence of movement in the period under study.

It is also instructive to contrast our estimates of the average cost of capital with those that would be obtained by following the prescriptions laid down in much of the traditional literature of corporation finance. Essentially, these call for computing the weighted sum of the market yields of each type of security, the weights being the “target” proportions of each security in the capital structure. That is, if we let $i =$ the earnings/price ratio (our $\bar{x}/S$); $p =$ the preferred dividend yield (our $\bar{P}/\bar{P}_A$); $r =$ the average rate of interest on bonds (our $\bar{R}/D$); $l =$ the target debt ratio; and $l' =$ the target preferred ratio then the weighted average cost of capital function under the traditional view can be expressed as $i(1-l-l') + p(l') + r(1-r)(l)$. Where the target weights $l$ and $l'$ are computed at book value as is usually recommended (i.e., with $l = D/A$ and $l' = P/A$ and $(1-l-l') = B/A$ in our notation) we shall refer to the resulting average as $C_B(D/A)$; where they are taken at market value (i.e., with $l = D/V$ and $l' = P/V$ and $1-l-l' = S/V$) we
shall refer to the average as \( \overline{C}_M(D/V) \), with unbarred values of the argument standing as before for a single company value and barred values for industry means. Estimates of both \( \overline{C}_B(D/V) \) and \( \overline{C}_M(D/A) \) for the typical firm in the sample, using actual sample mean values of \( i, p, \) and \( r \), as well as of the book- and market-value measures of \( l \) and \( l' \) in each case, are shown in blocks 5 and 6 of Table 10.

As can be seen from Table 10, both the levels and the time paths of \( \overline{C}_B(D/A) \) and \( \overline{C}_M(D/V) \) differ significantly from those of \( C(D/A) \). The largest discrepancies arise in the case of the widely used \( \overline{C}_B(D/A) \) measure which is substantially below \( C(D/A) \) in all three years and which shows only a very slight rise over the period. The market value estimates, \( \overline{C}_M(D/V) \) are considerably closer to those of \( C(D/A) \), but they too fail to indicate the sizable increase in the cost of capital which seems to have occurred during this period.

E. Reconciliation with Conventional Averages

To understand precisely why these three methods of estimating the average cost of capital gave such different answers for the years under study (and why they are likely to continue to diverge for other years and other industries), it is helpful to begin by showing how these estimates would relate to each other in a much simpler world in which no growth potential ever existed. In such a world, we have seen that the ratio \( (\overline{X}' - \tau \overline{R})/(V - \tau D) \) — the tax- and leverage-adjusted yield of the previous section — would be a measure of our \( \rho \), the cost of equity capital. Hence, from the standpoint of any individual firm, the \( C(L) \) function can be expressed as

\[
C(L) = \frac{\overline{X}' - \tau \overline{R}}{V - \tau D} (1 - \tau L) = \frac{\overline{X}' - \tau \overline{R}}{V} \cdot \frac{1 - \tau L}{1 - \tau D/V}.
\]

An alternative way of computing industry averages would be not to average the product of the mean yield times the mean weight as was done in Table 10, but to average the product of the yield and weight for each company separately. In the case of \( \overline{C}_M(D/V) \) this leads to the very simple relation

\[
\overline{C}_M(D/V) = \left( \frac{\overline{X}'}{V} \cdot S/V \right) + \left( \frac{P_d \sigma}{P} \cdot P/V \right) + \left( \frac{\overline{R}(1 - \tau)}{D} \cdot D/V \right)
\]

\[
= \frac{\overline{X}'}{V} + \frac{\overline{P}d_0}{V} + \frac{\overline{R}(1 - \tau)}{V} = \frac{\overline{X}' - \tau \overline{R}}{V}.
\]

The sample values of the latter yield are .038, .043, and .042, respectively, for 1954, 1956, and 1957. The (slight) difference between the two estimates arises, of course, from the fact that the company values of \( \overline{X}'/S \) are not independent of the values of \( S/V \), i.e., of the company’s degree of leverage.

Note also, for simplicity, we shall here and hereafter ignore the small difference between the book value and the market value of debt and preferred, using the market values in all cases.
The weighted average cost of capital function with market-value weights is

\[ \bar{C}_M(D/V) = \frac{\bar{\pi}^*}{S} \cdot \frac{S}{V} + \frac{\bar{P}_D}{P} \cdot \frac{P}{V} + \frac{\bar{R}(1 - \tau)}{D} \cdot \frac{D}{V} = \frac{\bar{X}^* - \tau \bar{R}}{V} \]  

and with the book-value weights,

\[ \bar{C}_B(D/A) = \frac{\bar{\pi}^*}{S} \cdot \frac{B}{A} + \frac{\bar{P}_D}{P} \cdot \frac{P}{A} + \frac{\bar{R}(1 - \tau)}{D} \cdot \frac{D}{A}. \]

Notice first that for the special case of \( L = D/V \)—i.e., when the target leverage coincides with current leverage at market value, \( D/V \), the function \( C(L) \) takes the value

\[ C(D/V) = \frac{\bar{X}^* - \tau \bar{R}}{V} = \bar{C}_M(D/V). \]

In other words, if a firm’s current and future target leverage is \( D/V \) it will get precisely the same estimate for its average cost of capital regardless of whether it chooses to multiply its current tax- and leverage-adjusted yield by \([1 - \tau(D/V)]\); or to compute the weighted average of the current yields of its outstanding securities, with market-value weights for each; or if it simply uses the ratio of expected tax-adjusted earnings to total market value. A similar equivalence of estimates (at least to a very close degree of approximation) would also hold, of course, for economists concerned with “typical” values for the industry and using industry mean values of \( D/V \) and of the various yields, i.e., \( C(D/V) \cong \bar{C}_M(D/V) \).

Note also that if \( V = A \)—which would tend to be the case if there were no growth past or future—then \( \bar{C}_B(D/A) \) becomes the same function as \( \bar{C}_M(D/V) \) and, by extension, as \( C(L) \). In this special case of no growth, therefore, all three company and industry-wide estimates will coincide.

This simple picture changes quite drastically, however, as soon as growth potential is introduced. The function \( C(L) \) must now be expressed as

\[ \text{\footnotesize 87 Although the equivalence holds for individual company data and for industry averages, there is one important case in which the equivalence very definitely does not hold. This is the common case of the firm following the weighted average approach of (23) with current (or prospective future) company weights, but using industry-wide averages of the component yields so as to obtain less noisy estimates. The trouble here is that the market yield on shares (and to some extent the yields on preferred and bonds as well) are increasing functions of leverage. Hence, for a firm whose target leverage is greater (smaller) than the average for the industry the industry mean yield will be an underestimate (overestimate) of its own yield and the resulting average cost of capital will be too low (high). This problem does not arise under our (22), of course, since \((\bar{X}^* - \tau \bar{R})/(V - \tau D)\) is not a function of firm policy (as is \( \bar{\pi}^*/S \)), but an estimate of the external, market-given parameter, \( \rho \).} \]
\begin{equation}
(25) \quad C(L) = \frac{\bar{X} - \tau \bar{R}}{V - \tau D - G} (1 - \tau L) = \frac{\bar{X} - \tau \bar{R}}{V - G} \cdot \frac{1 - \tau L}{1 - \tau \left( \frac{D}{V - G} \right)},
\end{equation}

where $G$ is the market's current valuation of future growth potential. Hence, as can be seen by reference back to (23), there no longer exists any concept of $L$ for which the function $C(L)$ will be the same as $\bar{C}_M(D/V)$. Note also that in the special case in which future growth potential constitutes the only major source of divergence between $V$ and $A$, $(\bar{D}/\bar{A}) \approx \left[ \frac{D/(V-G)}{\bar{A}} \right]$, so that our estimates of the average cost of capital $C(\bar{D}/\bar{A})$ would be closely approximated by the ratio $[(\bar{X} - \tau \bar{R})/(V-G)]$. The actual sample mean values of that ratio (with $G$ taken as the product of the growth coefficient in Table 3 and the mean value of our growth variable $\Delta A/A$) are shown in the last block of Table 10. As can be seen, the approximation to $C(\bar{D}/\bar{A})$ is indeed quite close in 1956 and 1957; but it is less satisfactory in 1954 where the growth contribution is small both in absolute terms and relative to the other sources of divergence between $V$ and $A$.

Where the ratio $[(\bar{X} - \tau \bar{R})/(V-G)]$ is a good approximation to $C(\bar{D}/\bar{A})$, it will, of course, also follow that both measures will exceed $\bar{C}_M(\bar{D}/\bar{V})$, which, as we saw above, is given approximately by $[(\bar{X} - \tau \bar{R})/\bar{V}]$. As for the relation between the popular $\bar{C}_B(\bar{D}/\bar{A})$ and $C(\bar{D}/\bar{A})$ note that we can express the ratio $[(\bar{X} - \tau \bar{R})/(V-G)]$ approximately as

\begin{equation}
(26) \quad \frac{\bar{X} - \tau \bar{R}}{V - G} = \left( \frac{\bar{\pi} \cdot S - G}{S - G \cdot V - G} \right) + \left( \frac{P_d \cdot P}{P \cdot V - G} \right)
\end{equation}

\begin{equation}
+ \left( \frac{\bar{\pi} \cdot (1 - \tau) \cdot D}{D \cdot V - G} \right) \approx \left( \frac{\bar{\pi} \cdot S - G}{\bar{A}} \right) \frac{B}{A},
\end{equation}

\begin{equation}
+ \left( \frac{P_d \cdot P}{P \cdot A} \right) + \left( \frac{\bar{\pi} \cdot (1 - \tau) \cdot D}{D \cdot A} \right),
\end{equation}

since the assumption $V - G \approx A$ implies

\begin{equation}
\frac{D}{V - G} \approx \frac{D}{A}, \quad \frac{P}{V - G} \approx \frac{P}{A} \quad \text{and} \quad \frac{S - G}{V - G} \approx \frac{B}{A}.
\end{equation}

Comparison with (24) shows that the weights in the two expressions are essentially the same; but since $(\bar{\pi}/S) < [\bar{\pi}/(S-G)]$, $\bar{C}_B(\bar{D}/\bar{A})$ too will necessarily fall short of $C(\bar{D}/\bar{A})$ when growth is present, and the gap will be larger, the larger is the contribution of growth to the value of the shares.
Once again, then, we see that attempts to infer the cost of capital directly from market yields rather than by the more detailed, cross-sectional estimating procedures developed in this paper break down in the face of growth. Where growth is present all of the popular, short-cut approximations will underestimate the cost of capital; and, where the market changes its evaluation of growth potential over time (as is inevitable in view of the nature of growth) the time path of the yield measures may give a quite misleading picture of the true changes in the cost of capital. In particular, in our sample it happens that the market's evaluation of growth increased substantially over the period, thereby causing the yield measures to understate seriously the rise in capital costs that appears to have been taking place at the same time. As noted earlier, it is somewhat paradoxical that these two changes should have occurred simultaneously, since an increase in the cost of capital should tend to reduce what the market is willing to pay for given investment opportunities. We can perhaps throw some light on this paradox by taking a closer look at our growth coefficients and their implicit components.

F. A Further Analysis of the Valuation of Growth

As noted earlier (cf. Sec. II.E above) the growth term in our basic valuation equation is of the form

\[ k\bar{X}(1 - \tau) \left[ \frac{\rho^* - C}{C(1 + C)} \right] T, \]

where \( k \) is the ratio of investment to tax-adjusted earnings; \( C \) = the average cost of capital; \( \rho^* \) = the tax-adjusted rate of return on new investment; and \( T \) is a measure of the length of time for which the opportunities to invest at the rate \( \rho^* \) are expected to last. In the actual estimating equations we have taken as our growth variable an estimate of \( k\bar{X}(1 - \tau) \), the level of future investment opportunities. Hence, if one accepts the underlying model, the observed coefficients of the growth variable can be interpreted as an approximation to

\[ [(\rho^* - C)/(C(1 + C))]T. \]

Now that we have estimates of \( C \), the average cost of capital for a typical firm, we can attempt some further decomposition of these growth coefficients.

In particular, it should be possible from what we know about past earnings and about the regulatory process governing earnings in the industry to make at least a rough approximation of \( \rho^* \).

An obvious first candidate as an approximation to \( \rho^* \) is, of course, the current, tax-adjusted rate of return on assets, \( (\bar{X} - \tau\bar{R})/A \). Such a
Table 11—Analysis of the Growth Effect

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth Coefficient* (C)</th>
<th>Average Cost of Capitalb C(D/A)</th>
<th>Average Tax-Adjusted Return on Assets ((\rho^*))</th>
<th>Average Return Assuming 6 per cent Return after Taxes ((\rho^*))</th>
<th>Implied Value of (T^*) (in years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>1.36</td>
<td>.046</td>
<td>.047</td>
<td>.052</td>
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<td>1956</td>
<td>.90</td>
<td>.045</td>
<td>.048</td>
<td>.052</td>
<td>12</td>
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<tr>
<td>1954</td>
<td>.30</td>
<td>.036</td>
<td>.046</td>
<td>.052</td>
<td>1</td>
</tr>
</tbody>
</table>

a From Table 3, Panel 2.
b From Table 10, Column 1.
computed as \(G((C(1+C)/(\rho^*-C))\).

measure, however, is almost certainly an underestimate of \(\rho^*\), since we know that there are components of total assets—actually, of total liabilities—that regulatory commissions systematically exclude from the rate base. The idea of the extent of this underestimate is provided by the knowledge that during these years most of the state commissions were still setting the "reasonable return on the rate base" in the near neighborhood of the classical 6 per cent. By contrast, the sample average values of \(\bar{X}_r/A - \bar{X}_r\), rather than \((\bar{X}_r - \bar{R})\), being the relevant earnings concept in rate-setting—were only .054, .056, and .055 per cent in 1954, 1956, and 1957, respectively. One simple adjustment, therefore, would be to blow up each sample mean value of \((\bar{X}_r - \bar{R})/A\) by the ratio of 6 per cent to the sample mean value of \(\bar{X}_r/A\). The rates of return so adjusted, as well as the original unadjusted rates of return, are presented in Table 11 along with the estimates of \(T^*\) they imply.

These results would seem to suggest the following as the resolution of the paradox described in the previous section. The observed rise in the markets' valuation of the industry's growth potential, in the face of the sharp rise in the cost of capital during the period, cannot reasonably be attributed to any compensating increase in the expected rate of return on future investment. No sharp upward trend in earnings rates, adjusted or unadjusted, is visible in the data; nor would such a trend be

A further word of caution is necessary because the so-called accounting rate of return (earnings after depreciation divided by net assets) is not the same as the ordinary internal rate of return when a firm is growing. This discrepancy does not seem likely to create any very serious problems insofar as the valuation equations or estimates of the cost of capital are concerned since in those equations assets appear only as a deflator and since an explicit growth variable is included. It may, however, raise difficulties for comparisons of the kind being attempted here. We say may, because the \(\rho^*\) in our formula is not the usual internal rate of return, but the so-called "perpetual rate of return" (see [14, p. 416]), and the relations between that rate and the accounting rate of return have, to our knowledge, nowhere yet been explored. We are indebted to Sidney Davidson and Robert Williamson for some helpful discussions on this general point.
expected in view of the regulatory controls over the level of earnings. What seems to have been happening rather is that early in the period investors came to recognize that the regulatory authorities were setting rates at levels that would probably permit firms in the industry to earn significantly more than the cost of capital on any new capital invested. The subsequent rise in the cost of capital narrowed the margin of gain somewhat; but its effects on valuation were more than offset by an increase in the length of time that favorable terms for new investment were expected to persist. The actual numerical estimates of this time horizon, as presented in Table 11, are not, of course, to be taken seriously in view of the many approximations, theoretical and empirical, involved in their computation. But a general expansion of horizons (or at least an increasing awareness of growth potential on the part of investors) is very definitely indicated.

Conclusion

This has been a long and necessarily somewhat tedious paper, and we do not propose to add either to the length or the tedium by an extensive summary at this point. Instead, we shall simply close by indicating briefly what we believe to be the main conclusions that can be drawn from the material presented. These are:

1. With respect to the methodology of empirical valuation and cost of capital studies, we feel we have demonstrated that the two-stage instrumental variable approach developed here can be an effective way of dealing with the problems caused by errors of measurement of expected future earnings—problems that bear a major share of the responsibility for the meager progress in empirical research on this front to date. We can also report some success in our attempt to provide an explicit measure of growth potential for the utility companies in our sample, but, clearly, more general methods of dealing with this crucial variable are urgently needed before substantial further progress on the cost of capital problem can be made.

2. With respect to the theory of valuation, we found that the rational-behavior, perfect-market model of valuation under uncertainty stands up quite well when confronted with the data both in terms of what it says should be included, and what it says should be excluded. In particular, for the utility industry in these years we found no evidence of sizable leverage or dividend effects of the kind assumed in much of the traditional literature of finance.

3. As for our estimates of the cost of equity capital and of the average cost of capital we showed that they differ very considerably both in level and movement during the sample period from the conventional
kinds of “yield” estimates so widely used in economics and finance. On the other hand, our estimates for the utility industry do seem to conform reasonably closely over the (short) sample period with movements in the long-term rate of interest on bonds. It will be interesting to see whether this relation persists in other industries and over longer and less placid intervals of time.

**Appendix A**

**Companies in the Sample**

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Company Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlantic City Electric Co.</td>
<td>Maine Public Service</td>
</tr>
<tr>
<td>Baltimore Gas &amp; Electric Co.</td>
<td>Minnesota Power &amp; Light</td>
</tr>
<tr>
<td>Boston Edison Co.</td>
<td>Missouri Public Service</td>
</tr>
<tr>
<td>Carolina Power and Light</td>
<td>Montana-Dakota Utilities Co.</td>
</tr>
<tr>
<td>Central Illinois Light</td>
<td>New York State Electric &amp; Gas Corp.</td>
</tr>
<tr>
<td>Central Illinois Public Service</td>
<td>Niagara Mohawk Power Corp.</td>
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<tr>
<td>Cincinnati Gas &amp; Electric Co.</td>
<td>Ohio Edison</td>
</tr>
<tr>
<td>Cleveland Electric Illuminating</td>
<td>Oklahoma Gas &amp; Electric</td>
</tr>
<tr>
<td>Columbus &amp; Southern Ohio Electric Co.</td>
<td>Pacific Gas &amp; Electric</td>
</tr>
<tr>
<td>Commonwealth Edison</td>
<td>Pennsylvania Power &amp; Light</td>
</tr>
<tr>
<td>Community Public Service</td>
<td>Public Service Co. of Colorado</td>
</tr>
<tr>
<td>Consolidated Edison of N. Y.</td>
<td>Puget Sound Power &amp; Light</td>
</tr>
<tr>
<td>Consumers Power</td>
<td>Rochester Gas and Electric Co.</td>
</tr>
<tr>
<td>Delaware Power and Light</td>
<td>San Diego Gas &amp; Electric</td>
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<tr>
<td>Duke Power</td>
<td>South Carolina Electric &amp; Gas Co.</td>
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<tr>
<td>Duquesne Light</td>
<td>Southern Indiana Gas &amp; Electric</td>
</tr>
<tr>
<td>Florida Power Corp.</td>
<td>Toledo Edison</td>
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<td>Florida Power and Light Co.</td>
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<td>Gulf States Utilities</td>
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<td>Public Service Electric &amp; Gas Co.</td>
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<td>Iowa Illinois Gas &amp; Electric</td>
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<td>Iowa Power &amp; Light</td>
<td>Houston Lighting &amp; Power Co.</td>
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<td>Potomac Electric Power Co.</td>
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<tr>
<td>Kansas Gas &amp; Electric Co.</td>
<td>Public Service Co. (Indiana)</td>
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<td>Southern California Edison Co.</td>
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<td>Long Island Lighting</td>
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### Appendix B

1. *Means and Standard Deviations of the Principal Variables*

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<th>1954</th>
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<td>.851</td>
<td>.869</td>
<td>.919</td>
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<tr>
<td>$A$</td>
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<td>(.11)</td>
<td>(.093)</td>
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<td>.0484</td>
<td>.0464</td>
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<td>(.006)</td>
<td>(.006)</td>
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<tr>
<td>$1/A \cdot 10^8$</td>
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(Appendix B continued on next page)
### Matrix of Simple Correlation Coefficients

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<td>$\frac{V - \tau D}{A}$</td>
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<tr>
<td>2.</td>
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<tr>
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<tr>
<td>4.</td>
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<table>
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<td>3.</td>
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<td>4.</td>
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<td>5.</td>
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<td>6.</td>
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<tr>
<td>7.</td>
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REFERENCES


