Problems to the lecture
„Physical Metallurgy“ („Materialkunde“)

Chapter 6: Mechanical Properties

WS 1999/2000

Explanation:

A-question: Solve the following question with the help of the lecture notes.
B-question: solved examples
C-question: Solve the following question on your own with the help of the lecture notes and the hints given in section B

1J=1Nm, 1Å=10^{-10} m, R=8.3144 J/molK,
k=8.62\cdot10^{-5} eV/K = 1.38\cdot10^{-23} J/K, N_L=6.023\cdot10^{23} mol^{-1}

Materials properties for Aluminium:
G=27\cdot10^9 N/m^2, a=4.04Å, T_S=660°C, \rho=2.7g/cm^3, A=27.0g/mol, \nu=0.34
\gamma_{SFE}=18.10^{-2} J/m^2, \gamma_{KG}=0.6J/m^2

Materials properties for Copper:
G=48\cdot10^9 N/m^2, a=3.61Å, T_S=1083°C, \rho=8.7g/cm^3, A=63.5g/mol, \nu=0.35
\gamma_{SFE}=5.10^{-2} J/m^2, \gamma_{KG}=0.5J/m^2, \gamma_{Ob}=1J/m^2
A-question:
(a) Sketch and label schematically the components of a stress tensor 2\textsuperscript{nd} rank.
(b) What is a stress deviator?
(c) Break up a general stress tensor 2\textsuperscript{nd} rank into its hydrostatic and deviatoric components (formulas).
(d) Sketch and explain Mohr’s circle.

C-question:
Give the general notation for a rotation matrix for the following cases:
(a) Rotation in 2-dimensional case.
(b) Rotation in 3-dimensional case around the x-axis, the y-axis and the z-axis respectively.
(c) How do you rotate a 1\textsuperscript{st} and a 2\textsuperscript{nd} rank tensor respectively?

C-question:
In an orthogonal and normalised co-ordinate system, perform three successive rotations around the x-axis (angle $\alpha$), the y-axis (angle $\beta$) and the z-axis (angle $\gamma$) in the 3-dimensional case. Calculate the formula for the resulting total rotation as a function of all three angles.

B-question:
In a 2-dimensional system the following stress tensor is given: $\sigma = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}$ 100 MPa
(a) Rotate this stress tensor $\sigma$ by 35°.
(b) Determine the main stress tensor. By how many degrees do you have to rotate $\sigma$?
(c) Calculate the maximum shear stress.

A-question:
What form does Hooke’s law take in the 1-dimensional and in tensorial notation? How many components does the tensor of the elastic constants comprise? Why is the number of components actually reduced for the most materials?

B-question:
Discuss the tensile test on a slender rod.
(a) Draw a qualitative diagram of the nominal stress over strain. Which material constants can be retrieved from this diagram?
(b) Draw a qualitative diagram of the true stress over strain.
(c) How can you deduce the diagrams a) and b) from each other?
(d) Explain the concept of „physical hardening“ and „geometrical softening“.
(e) Why does the sample finally break?

B-question:
Prove the validity of the following equation for the end of the uniform elongation in tensile tests (Considère-construction): $\frac{d\sigma}{d\varepsilon} = \frac{\sigma}{1 + \varepsilon}$.
C-question:

a) Which vectors determine a dislocation? Draw a rough sketch.
b) How can one determine the slip plane by the use of these two vectors?
c) Explain the Burgers-circuit (first fix a co-ordinate system).
d) How many different types of dislocations exist? What is the difference between edge and screw dislocations with regard to their freedom of movement?
e) Which dislocations types does a dislocation ring situated in the slip plane consist of?
   In which direction do the individual dislocations move under external shear stress? (Draw a sketch of a cubic body. Draw the Burgers Vector parallel to one of the axis. Fix the line element of the dislocation.)
   What does the crystal look like when the dislocation ring has left the crystal?
f) Is it possible to produce a dislocation rings that consist exclusively of either edge or screw dislocations? Give an explanation.
   What does the crystal look like when a prismatic dislocation that consists exclusively of edge dislocations ring has left the crystal?
g) How do dislocation rings develop (mixed dislocation ring, prismatic dislocation ring)?
h) How many atoms must condense to let a prismatic dislocation ring with a radius of $r = 0.6 \mu m$ develop in a cubic primitive crystal ($a = 0.35 \text{ nm}$)?

B-question:

What is conservative and non-conservative dislocation motion?

B-question:

a) Deduce the tension necessary to bow a dislocation line that is fixed between two anchor points.
b) Deduce a expression for the activation of a Frank-Read source in dependence of tension. What is the significance of such sources?

A-question:

Name some typical slip systems of pure metals.

C-question:

What is the theoretical shear strength? Deduce a formula for calculating it. What is the approximate value of the theoretical shear strength for Au ($E_{\text{Au}} = 78 \text{ GPa}$, $\nu_{\text{Au}} = 0.44$)?

C-question:

What is the maximum possible dislocation density in a crystal? What is the dislocation length in 1 m³ of very high deformed copper (in light years)?

B-question:

What is the possible shear of an annealed copper single crystal (length of a side $l = 1 \text{ cm}$) if all dislocations leave the crystal during deformation ($\rho = 10^6 \text{ cm}^{-2}$)? Is this in agreement with the experience?
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**B-question:**
Bending of a sample produces a surplus of edge dislocations of one sign. Deduce a relation between the surplus dislocation density $\rho$ and the radius of the curvature $r$. What are these dislocations called?

**B-question:**
Calculate the shear during twinning in the
a) fcc lattice
b) and bcc lattice.

**C-question:**
a) Discuss deformation twinning in hexagonal metals with a $c/a$-ratio of $c/a < 1.73$ and $c/a > 1.73$ for tensile strain of a single crystal where the strain axis is parallel to the $c$-axis, where the strain axis is parallel to the basal plane and for the rolling deformation of a polycrystal.
b) Discuss deformation twinning in hexagonal lattice with a $c/a$-ratio of $c/a = 1.73$. Which consideration leads to the value of 1.73?

**B-question:**
Calculate for a screw dislocation:
 a) the shear stress at the distance $r$ from the dislocation core.
b) the density of energy and the total energy per length unit. Assume $\tau_{\text{core}} = G/(2\cdot\pi)$ for the energy of the dislocation core. Be the inner cut-off radius $b$.
c) Prove that the major part of the energy is in the elastic field.
d) On the basis of these results, discuss the freedom of movement of a dislocation again.

**B-question:**
With the help of the Peach-Koehler formula, calculate the force exerted by the shear stress field of an edge dislocation on another edge dislocation with a parallel Burgers vector. Draw a rough sketch.
 a) Discuss the course of the force between these two dislocations (stable positions, maximum shear stress).
b) Prove that for the yield stress the following relation is true: $\tau \sim \sqrt{\rho}$

**C-question:**
With the help of the Peach-Koehler formula, calculate the force exerted by the shear stress field of an edge dislocation on another edge dislocation with an anti-parallel Burgers vector. Draw a rough sketch. Discuss the course of the force between these two dislocations (stable positions, maximum shear stress).

**C-question:**
Calculate the force per length unit exerted by an edge dislocation parallel to the $z$-axis on another edge dislocation also parallel to the $z$-axis but with orthogonal Burgers vector. Which are the stable positions?
B-question:
Calculate the force between a parallel edge and screw dislocation.

C-question:
Calculate the force between two parallel screw dislocations.

C-question:
Three small-angle tilt boundaries are arranged at a distance of 20·b parallel to each other. They consist of 10 identical edge dislocations each (b parallel to the normal of the boundary, b oriented in direction of the x-axis, line element oriented in direction of the z-axis). Calculate the climbing force that is exerted by the two external grain boundaries on a single dislocation situated in the grain boundary in the centre. Assume that the stress field of the grain boundary can be equated with the stress field of a single dislocation with b_{KWK} = 10·b. What does the stress field of a small-angle tilt boundary of infinite extension look like in ideal case?

B-question:
A rod clamped in on one side contains one edge dislocation (sketch). In which direction will the dislocation move under the applied bending force F? How will the rod look like after the deformation?

C-question:
a) Calculate the force that is exerted on the dislocation in the sketch below. How will the dislocation move under the influence of this force. Does any temperature dependence of the dislocation movement exist under this condition? The line element is parallel to +z.
b) Calculate the force that is exerted on the dislocation in the sketch below. How will the dislocation move under the influence of this force. Does any temperature dependence of the dislocation movement exist under this condition? The line element is parallel to +z.

\[ \begin{array}{c}
\text{x} \\
\text{y} \\
\text{z}
\end{array} \]

F

B-question:
\[ \text{a) Deduce a formula for calculating the Schmid factor.} \]
\[ \text{b) Name Schmid's law.} \]

B-question:
The axis of a tensile specimen of a copper single crystal is parallel to the [236]-direction.
\[ \text{a) Determine all possible slip systems and calculate the Schmid factor for each.} \]
\[ \text{b) Which slip system will be active first? For plastic deformation, which tensile load has to be applied (} \tau_{\text{krit}} = 3 \text{ MPa)?} \]
\[ \text{c) How will the tensile axis move during further plastic deformation?} \]
\[ \text{d) Determine the cross slip system.} \]
\[ \text{e) Explain this tensile test, if the axis of the tensile specimen is originally orientated parallel to [001].} \]

C-question:
The axis of a (i) tensile specimen of a Fe single crystal is parallel to the \( \begin{array}{c}
\text{124}
\end{array} \)-direction, (ii) the axis of a compression specimen of a Cu single crystal is parallel to the [236]-direction, and (iii) the axis of a compression specimen of a Fe single crystal is parallel to the \( \begin{array}{c}
\text{124}
\end{array} \)-direction.
\[ \text{Explain for each case the following aspects:} \]
\[ \text{a) Determine all possible slip systems and calculate the Schmid factor for each.} \]
\[ \text{b) Which slip system will be active first? For plastic deformation, which tensile / compressive load has to be applied (} \tau_{\text{krit}} = 3 \text{ MPa)?} \]
\[ \text{c) How will the specimen axis move during further plastic deformation?} \]
\[ \text{d) Determine the cross slip system.} \]
\[ \text{e) Explain this tensile / compression test, if the axis of the specimen is originally orientated parallel to [001].} \]
C-question:
a) A composite material consists of two single crystals in a row, Cu and Fe. The Cu single crystal has a [123]-orientation and the Fe single crystal has a [124]-orientation. Determine for both crystals the slip system with the maximum resolved shear stress in the case of a tensile test and in case of a compression test. Name the conjugate slip system. Calculate the Schmid factor for the active slip systems.
b) Which tensile load $\sigma$ has to be applied for plastic deformation ($\tau_{\text{krit(Cu)}} = 3$ MPa, $\tau_{\text{krit(Fe)}} = 4$ MPa)?
c) Which tensile load $\sigma$ has to be applied for plastic deformation, if both single crystals are arranged side by side ($\tau_{\text{krit(Cu)}} = 3$ MPa, $\tau_{\text{krit(Fe)}} = 4$ MPa)?

C-question:
The axis of a tensile specimen of a NiAl single crystal (B2 structure) is parallel to the [123]-direction. Assume that crystallographic slip takes place on {100}-planes in <100>-direction.
a) Determine all possible slip systems and calculate the Schmid factor for each.
b) Which slip system will be active first? For plastic deformation, which tensile load has to be applied for plastic deformation ($\tau_{\text{crit}} = 5$ MPa)?
c) How will the tensile axis move during further plastic deformation?
d) Explain this tensile test, if the axis of the tensile specimen is originally orientated parallel to $[\bar{1}1\bar{1}]$.

C-question:
For calculating a $\tau$-$\gamma$ curve from the $\sigma$-$\varepsilon$ data the change of the Schmid factor during deformation must be taken into account. Deduce the corresponding formula by the use of the following relations: $l/l_0 = 1+\varepsilon = \sin(\lambda_0)/\sin(\lambda) = \cos(\lambda_0)/\cos(\lambda)$ and $\tau\cdot d\gamma = \sigma\cdot d\varepsilon$.

A-question:
Discuss the $\tau$-$\gamma$-curves for tensile tests on Cu and Al at different temperatures and orientations. What is the main difference in the deformation behaviour of both metals? What are the characteristic parts of the $\tau$-$\gamma$ curve?

B-question:
a) Explain the dissociation of dislocations, which necessary condition has to be fulfilled.
b) Sketch the dissociation of a dislocation in the fcc lattice. What is the amount and the direction of the resulting partial dislocations?
c) Calculate the dissociation width in Cu and Al ($a_{\text{Cu}} = 3.6$ Å, $G_{\text{Cu}} = 4.8\cdot10^4$ N/mm$^2$, $\gamma_{\text{Cu}} = 0.05$ N/m, $a_{\text{Al}} = 4.0$ Å, $G_{\text{Al}} = 2.7\cdot10^4$ N/mm$^2$, $\gamma_{\text{Al}} = 0.18$ N/m).
d) What effect has the dissociation on the hardening rate?

A-question:
What is the main difference in the deformation behaviour between single and polycrystals (sketch)? How are the active slip systems determined for the single crystal, how are they determined for a polycrystal? What is the Taylor factor?
A-question:
a) Name the most important possibilities for increasing the strength of pure metals and alloys.
b) Which of these possibilities are suitable for applications at high homologous temperatures? Give an explanation. What are typical applications and materials for the use at high temperatures?

A-question:
a) Explain the different possible interactions of solid solution hardening. What are the basic differences in the cause and the result of these effects?
b) At a constant deviation of the modulus and the lattice parameter, what is the maximum possible strength? Name two examples of technical alloys with strong solid solution hardening.

B-question:
Deduce the development of strength if a straight edge dislocation passes an immobile foreign atom under the assumption of parelastic interaction (start with the interaction energy). Make a sketch.

C-question:
a) Deduce an expression for the increase of shear strength because of foreign atoms.
b) In the binary system Ni-Cu with complete miscibility the lattice constants are very similar. Parelastic interaction can therefore be neglected. Calculate the critical shear strength for Cu-5 atom% Ni at room temperature. (dlnG/dc_Ni = 0.62. Calculate F_{max} for y = 0.5·d_{(111)}. Assume that: Ω = b^3.)

A-question:
Explain the concept of dynamic strain aging. Give two examples.

A-question:
Discuss the Hall-Petch relationship.

B-question:
Give an equation for the Orowan critical stress. Deduce an expression for the relation between the spacing of particles, their radius and the precipitate volume fraction.

C-question:
Calculate the Orowan critical stress for a particle reinforced Cu sample (Al_2O_3, radius=r). (G_{cs} = 40 GPa, b_{cs} = 0.25 nm, b_{Al} = 0.29 nm, r = 100 nm, f = 1 Vol.%)
C-question:
a) Explain the concept of anelastic behaviour. Sketch the strain change for constant stress as a function of time.
b) Sketch the change of stress for constant strain as a function of time (σ-τ diagram) for an anelastic body.

A-question:
a) What are the positions (interstitial sites) of the C-atoms in α-Fe, how many possible positions exist per unit cell?
b) Sketch for the Snoek effect the temperature dependence of the damping for (i) three different frequencies and (ii) for three different ageing times.