Chapter 6:
Modulation Techniques for Mobile Radio

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Outline

- Frequency versus Amplitude Modulation
- Amplitude Modulation (AM)
- Angle Modulation
- Digital Modulation
- Line Coding
- Pulse Shaping Techniques
- Geometric Representation of Modulation Signal
- Linear Modulation Techniques
- Constant Envelope Modulation
- M-ary signaling

What is modulation

- **Modulation** is the process of encoding information from a message source in a manner suitable for transmission
- It involves translating a baseband message signal to a bandpass signal at frequencies that are very high compared to the baseband frequency.
- Baseband signal is called *modulating* signal
- Bandpass signal is called *modulated* signal

Modulation Techniques

- Modulation can be done by varying the
  - Amplitude
  - Phase, or
  - Frequency

  of a high frequency carrier in accordance with the amplitude of the message signal.
- **Demodulation** is the inverse operation: *extracting the baseband message from the carrier* so that it may be processed at the receiver.

Analog/Digital Modulation

- Analog Modulation
  - The input is continues signal
  - Used in first generation mobile radio systems such as AMPS in USA.
- Digital Modulation
  - The input is time sequence of symbols or pulses.
  - Are used in current and future mobile radio systems

Goal of Modulation Techniques

- Modulation is difficult task given the hostile mobile radio channels
  - Small-scale fading and multipath conditions.
- The goal of a modulation scheme is:
  - Transport the message signal through the radio channel with best possible quality
  - Occupy least amount of radio (RF) spectrum.

Frequency versus Amplitude Modulation

- Frequency Modulation (FM)
  - Most popular analog modulation technique
  - Amplitude of the carrier signal is kept constant (constant envelope signal), the frequency of carrier is changed according to the amplitude of the modulating message signal; Hence info is carried in the **phase** or **frequency** of the carrier.
  - Has better noise immunity:
    - atmospheric or impulse noise cause rapid fluctuations in the amplitude of the received signal
  - Performs better in multipath environment
    - Small-scale fading cause amplitude fluctuations as we have seen earlier.
  - Can trade bandwidth occupancy for improved noise performance.
    - Increasing the bandwith occupied increases the SNR ratio.
  - The relationship between received power and quality is non-linear.
    - Rapid increase in quality for an increase in received power.
    - Resistant to co-channel interference (capture effect).

Frequency versus Amplitude Modulation

- Amplitude Modulation (AM)
  - Changes the amplitude of the carrier signal according to the amplitude of the message signal.
  - All info is carried in the amplitude of the carrier
  - There is a linear relationship between the received signal quality and received signal power.
  - AM systems usually occupy less bandwidth then FM systems.
  - AM carrier signal has time-varying envelope.

Amplitude Modulation

- The amplitude of high-carrier signal is varied according to the instantaneous amplitude of the modulating message signal m(t).

Carrier Signal : $A_c \cos(2pf_c t)$

Modulating Message Signal : $m(t)$

The AM Signal : $s_{AM}(t) = A_c[1 + m(t)]\cos(2pf_c t)$

Modulation Index of AM Signal

For a sinusoidal message signal $m(t) = A_m \cos(2pf_m t)$

Index is defined as: $$k = \frac{A_m}{A_c}$$

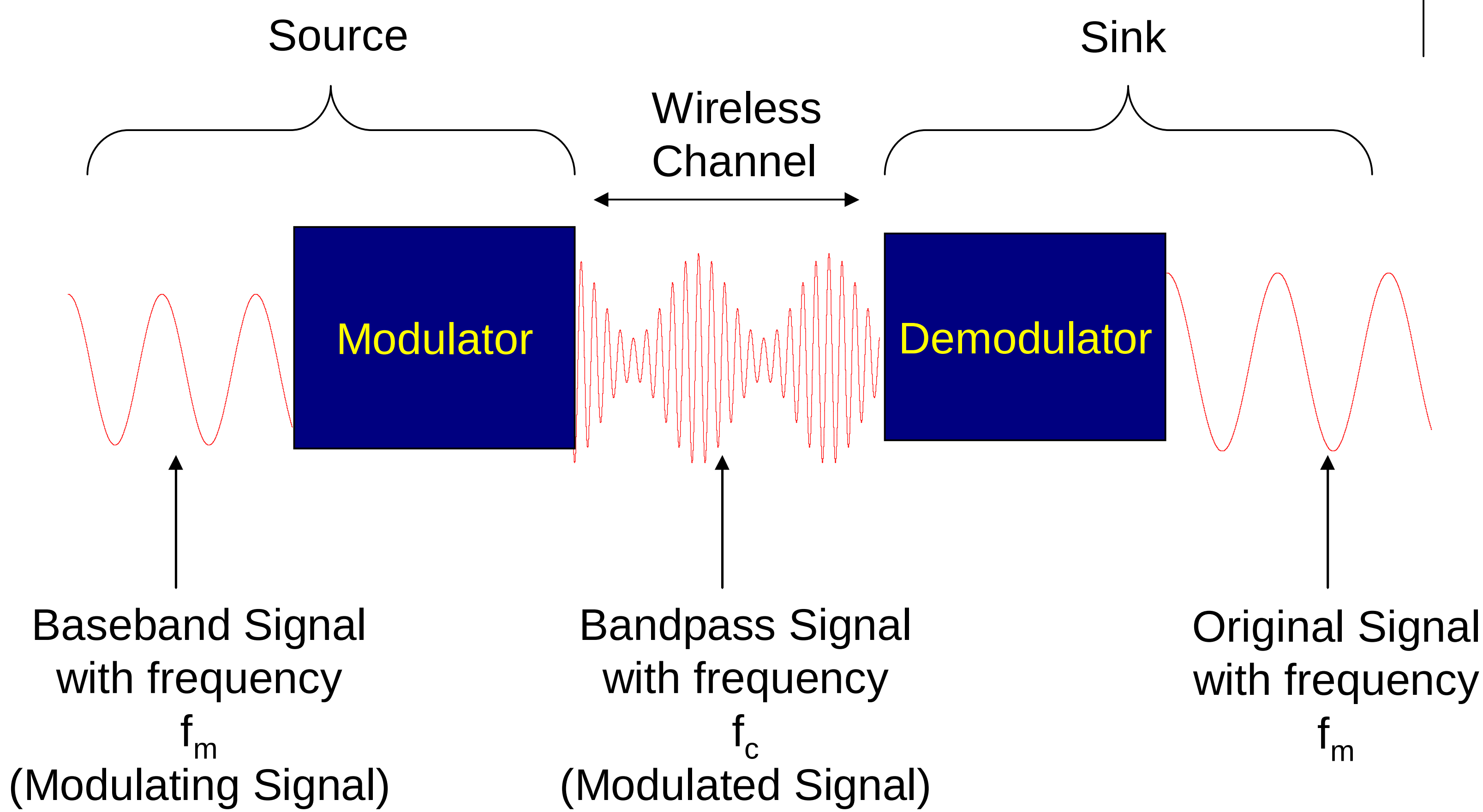
$S_{AM}(t)$ can also be expressed as:

$$s_{AM}(t) = \text{Re}\{g(t)e^{j2pf_c t}\}$$

where $g(t) = A_c(1 + m(t)]$

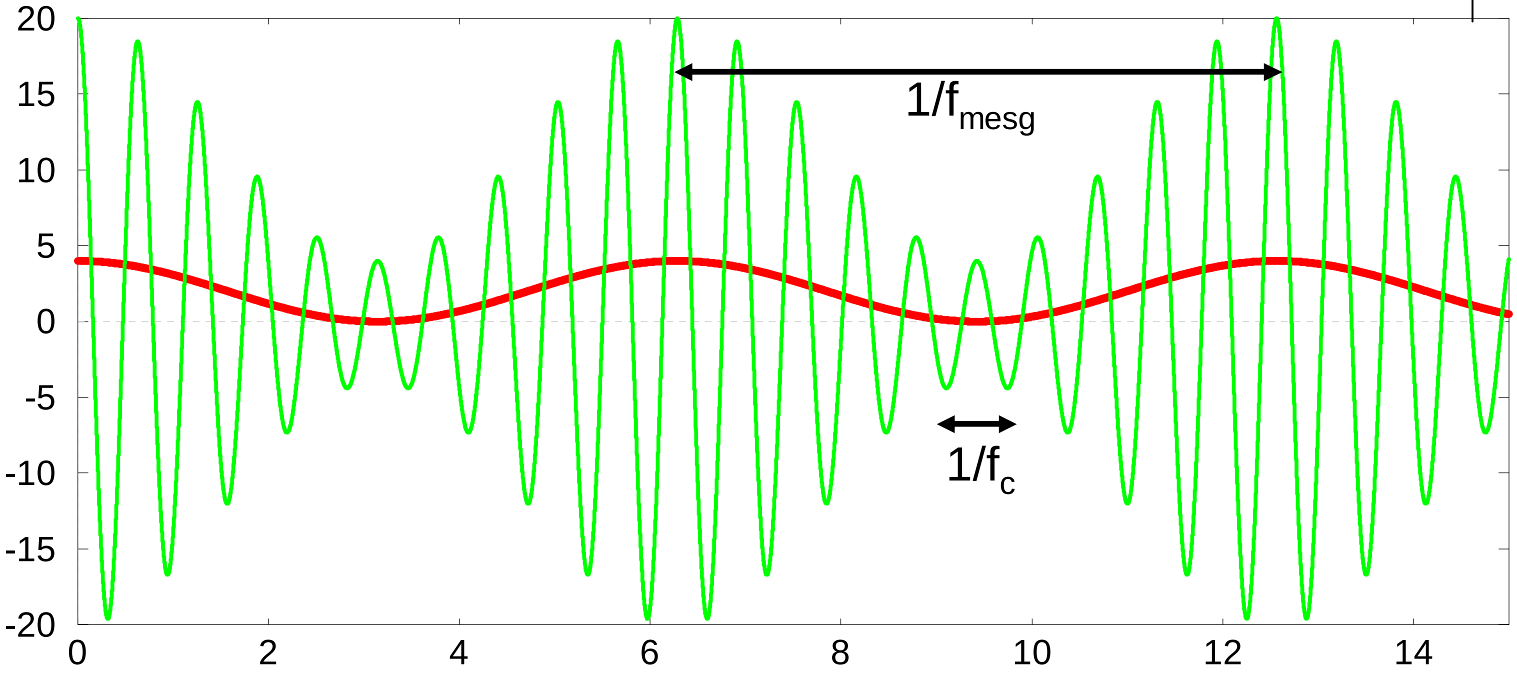
g(t) is called the complex envelope of AM signal.

AM Modulation/Demodulation



$f_c >> f_m$

AM Modulation - Example



Message signal : $m(t) = 2 + 2\cos(t)$

Carrier signal : $A_c \cos(2pf_c t) = 4\cos(10t)$

$s_{AM}(t) = A_c[1 + m(t)]\cos(2pf_c t)$

AM Signal : $s_{AM}(t) = 4[1 + 2 + 2\cos(t)]\cos(2pf_c t)$

$$f_c = \frac{10}{2p} \cong 1.6 Hz$$

$$f_{mesg} = \frac{1}{2p} = 0.16 Hz$$

Angle Modulation

- Angle of the carrier is varied according to the amplitude of the modulating baseband signal.
- Two classes of angle modulation techniques:
  - Frequency Modulation
    - Instantaneous frequency of the carrier signal is varied linearly with message signal m(t)
  - Phase Modulation
    - The phase $\theta(t)$ of the carrier signal is varied linearly with the message signal m(t).

Angle Modulation

FREQUENCY MODULATION

$$s_{FM}(t) = A_c \cos(2pf_c t + q(t)] = A_c \cos\left[2pf_c t + 2pk_f \int_{-\infty}^{t} m(x)dx\right]$$

k_f is the frequency deviation constant (kHz/V)

If modulation signal is a sinusoid of amplitude A_m, frequency f_m:

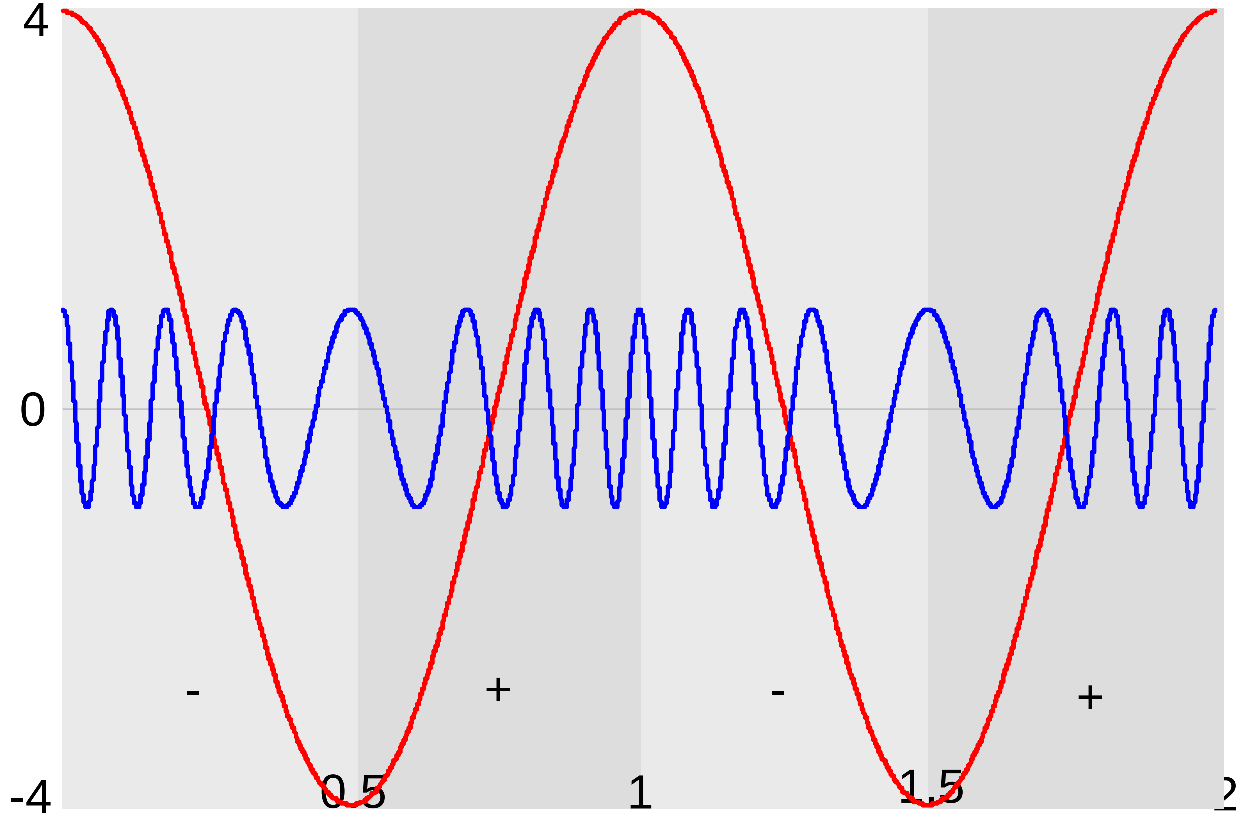
$$s_{FM}(t) = A_c \cos(2pf_c t + \frac{k_f A_m}{f_m} \sin(2pf_m t)]$$

PHASE MODULATION

$$s_{PM}(t) = A_c \cos\left[2pf_c t + k_q m(t)\right]$$

k_θ is the phase deviation constant

FM Example:



Message signal $m(t) = 4\cos(2pt)$

FM Signal $s(t) = \cos\left[2p\,8t + 4\sin(2pt)\right]$

Carrier Signal $\cos(2p\,8t)$

FM Index

$$b_f = \frac{k_f A_m}{W} = \frac{\Delta f}{W}$$

W: the maximum bandwidth of the modulating signal
Δf: peak frequency deviation of the transmitter.
A_m: peak value of the modulating signal

Example: Given $m(t) = 4\cos(2\pi 4\times 10^3 t)$ as the message signal and a frequency deviation constant gain (k_f) of 10kHz/V; Compute the peak frequency deviation and modulation index!

Answer: f_m=4kHz

Δf = 10kHz/V * 4V = 40kHz.
β_f = 40kHz / 4kHz = 10

Spectra and Bandwidth of FM Signals

An FM Signal has 98% of the total transmitted power in a RF bandwidth B_T

Carson's Rule

$$B_T = 2(b_f + 1) f_m$$ Upper bound

$$B_T = 2\Delta f$$ Lower bound

Example:

Analog AMPS FM system uses modulation index of B_f = 3 and f_m = 4kHz.

Using Carson's Rule: AMPS has 32kHz upper bound and 24kHz lower bound on required channnel bandwidth.

FM Demodulator

- Convert from the frequency of the carrier signal to the amplitude of the message signal
- FM Detection Techniques
  - Slope Detection
  - Zero-crossing detection
  - Phase-locked discrimination
  - Quadrature detection

Slope Detector

$$v_1(t) = V_1\cos\left[2pf_c t + q(t)\right] = V_1\cos\left[2pf_c t + 2pk_f\int_{-\infty}^{t} m(x)dx\right]$$

$$v_2(t) = \frac{dv_1(t)}{dt} = -V_1\left[2pf_c + \frac{dq}{dt}\right]\sin\left(2pf_c t + q(t)\right)$$

$$v_{out}(t) = V_1\left[2pf_c + \frac{d}{dt}q(t)\right]$$

$$= V_1 2pf_c + V_1 2pk_f m(t)$$

← Proportional to the priginal Message Signal

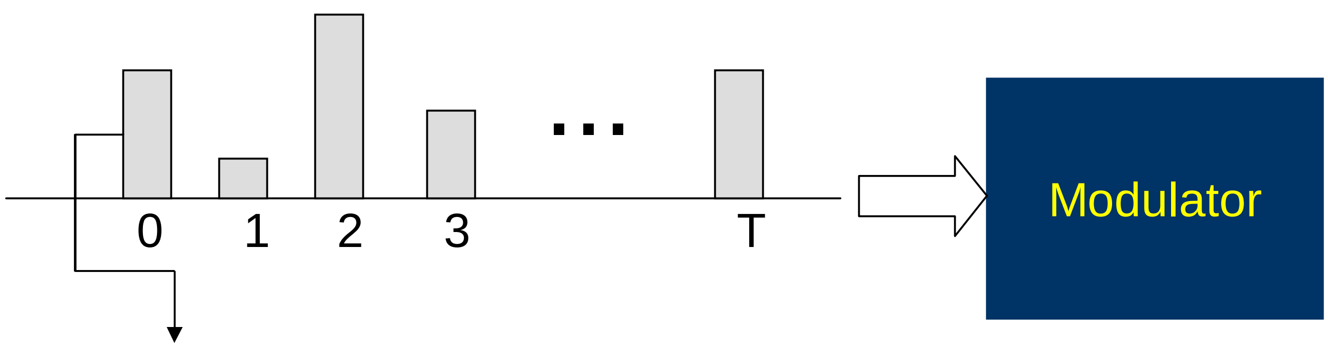
Digital Modulation

- The input is discrete signals
  - Time sequence of pulses or symbols
- Offers many advantages
  - Robustness to channel impairments
  - Easier multiplexing of variuous sources of information: voice, data, video.
  - Can accommodate digital error-control codes
  - Enables encryption of the transferred signals
    - More secure link

Digital Modulation

The modulating signal is respresented as a time-sequence of symbols or pulses.

Each symbol has *m* finite states: That means each symbol carries *n* bits of information where $n = \log_2 m$ bits/symbol.



One symbol
(has *m* states – voltage levels)
(represents $n = \log_2 m$ bits of information)

Factors that Influence Choice of Digital Modulation Techniques

- A desired modulation scheme
  - Provides low bit-error rates at low SNRs
    - Power efficiecny
  - Performs well in multipath and fading conditions
  - Occupies minimum RF channel bandwidth
    - Bandwidth efficieny
  - Is easy and cost-effective to implement
- Depending on the demands of a particular system or application, tradeoffs are made when selecting a digital modulation scheme.

Power Efficiency of Modulation

- Power efficiency is the ability of the modulation technique to preserve fidelity of the message at low power levels.
- Usually in order to obtain good fidelity, the signal power needs to be increased.
  - Tradeoff between fidelity and signal power
  - Power efficiency describes how efficient this tradeoff is made

$$\text{Power Efficiency}: \eta_p = \left[\frac{E_b}{N_0} \text{ required at the receiver input for certain } PER \right]$$

E_b: signal energy per bit
N_0: noise power spectral density
PER: probability of error

Bandwidth Efficiency of Modulation

- Ability of a modulation scheme to accommodate data within a limited bandwidth.
- Bandwidth efficiency reflect how efficiently the allocated bandwidth is utilized

$$\text{Bandwidth Efficiency}: \eta_B = \frac{R}{B} \text{ bps/Hz}$$

R: the data rate (bps)
B: bandwidth occupied by the modulated RF signal

Shannon's Bound

There is a fundamental upper bound on achievable bandwidth efficiency. Shannon's theorem gives the relationship between the channel bandwidth and the maximum data rate that can be transmitted over this channel considering also the noise present in the channel.

Shannon's Theorem

$$h_{B\max} = \frac{C}{B} = \log_2\left(1 + \frac{S}{N}\right)$$

C: channel capacity (maximum data-rate) (bps)
B: RF bandwidth
S/N: signal-to-noise ratio (no unit)

Tradeoff between BW Efficiency and Power Efficiency

- There is a tradeoff between bandwidth efficiency and power efficiency
  - Adding error control codes
    - Improves the power efficiency
      - Reduces the requires received power for a particular bit error rate
    - Decreases the bandwidth efficiency
      - Increases the bandwidth occupancy
  - M-ary keying modulation
    - Increases the bandwidth efficiency
    - Decreases the power efficiency
      - More power is requires at the receiver

Example:

- SNR for a wireless channel is 30dB and RF bandwidth is 200kHz. Compute the theoretical maximum data rate that can be transmitted over this channel?
- Answer:

$$\frac{S}{N} = 10^{\left[\frac{30\,dB}{10}\right]}$$

$$C = B\log_2(1+\frac{S}{N}) = 2x10^5 \log_2(1+1000) = \boxed{1.99Mbps}$$

Noiseless Channels and Nyquist Theorem

For a noiseless channel, Nyquist theorem gives the relationship between the channel bandwidth and maximum data rate that can be transmitted over this channel.

Nyquist Theorem

$$C = 2B\log_2 m$$

C: channel capacity (bps)
B: RF bandwidth
m: number of finite states in a symbol of transmitted signal

Example: A noiseless channel with 3kHz bandwidth can only transmit maximum of 6Kbps if the symbols are binary symbols.

Power Spectral Density (PSD) of Digital Signals and Bandwidth

- What does signal bandwidth mean?
- Answer is based on Power Spectral Density (PSD) of Signals
- For a random signal w(t), PSD is defined as:

$$Pw(f) = \lim_{T \to \infty} \left(\frac{\overline{|W_T(f)|^2}}{T} \right)$$

$W_T(f)$ is th fourier transform of $w_T(t)$

$$w_T(t) = \begin{cases} w(t) & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

Fourier Analysis

Joseph Fourier has shown that any periodic function F(f) with period T, can be constructed by summing a (possibly infinite) number of sines and cosines.

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n w_T t) + b_n \sin(n w_T t)$$

$$w_T = \frac{2p}{T} = 2p f_0$$

Such a decomposition is called *Fourier series* and the coefficients are called the *Fourier coefficients*.

A line graph of the amplitudes of the Fourier series components can be drawn as a function of frequency. Such a graph is called a *spectrum* or frequency spectrum. f_0 is called the *fundemantal frequency.*

The n^{th} term is called *n^{th} harmonic*. The coefficients of the n^{th} harmonic are a_n and b_n.

Fourier Analysis

The coefficients can be obtained from the periodic function F(t) as follows:

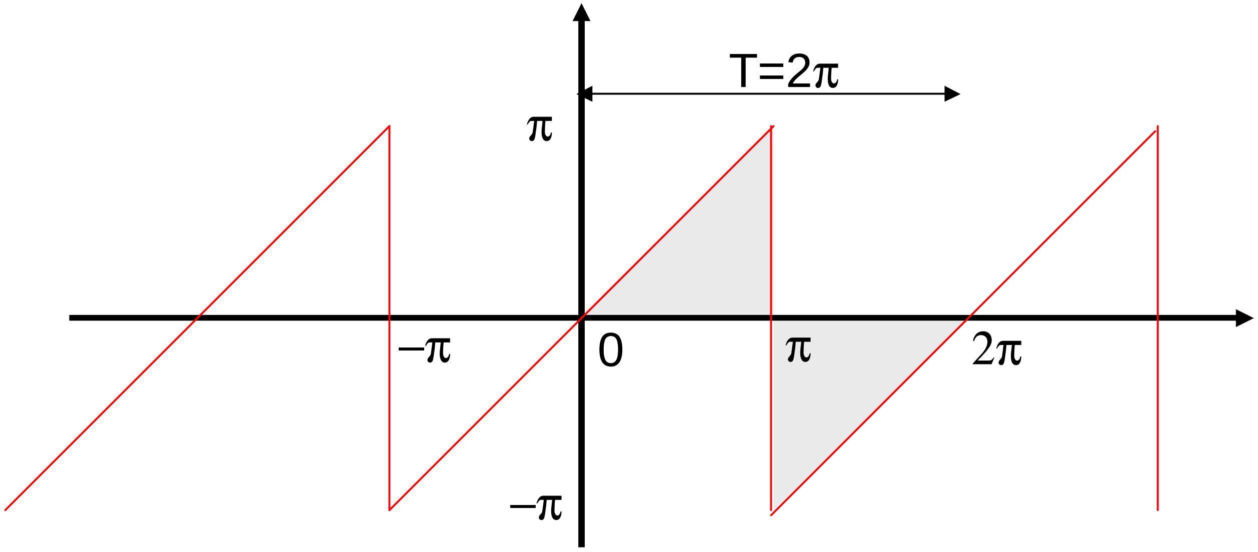
$$a_0 = \frac{2}{T}\int_0^T F(t)dt$$

$$a_n = \frac{2}{T}\int_0^T F(t)\cos n\nu_T t dt, \qquad n = 1,2,...$$

$$b_n = \frac{2}{T}\int_0^T F(t)\sin n\nu_T t dt, \qquad n = 1,2,...$$

Example: A Periodic Function

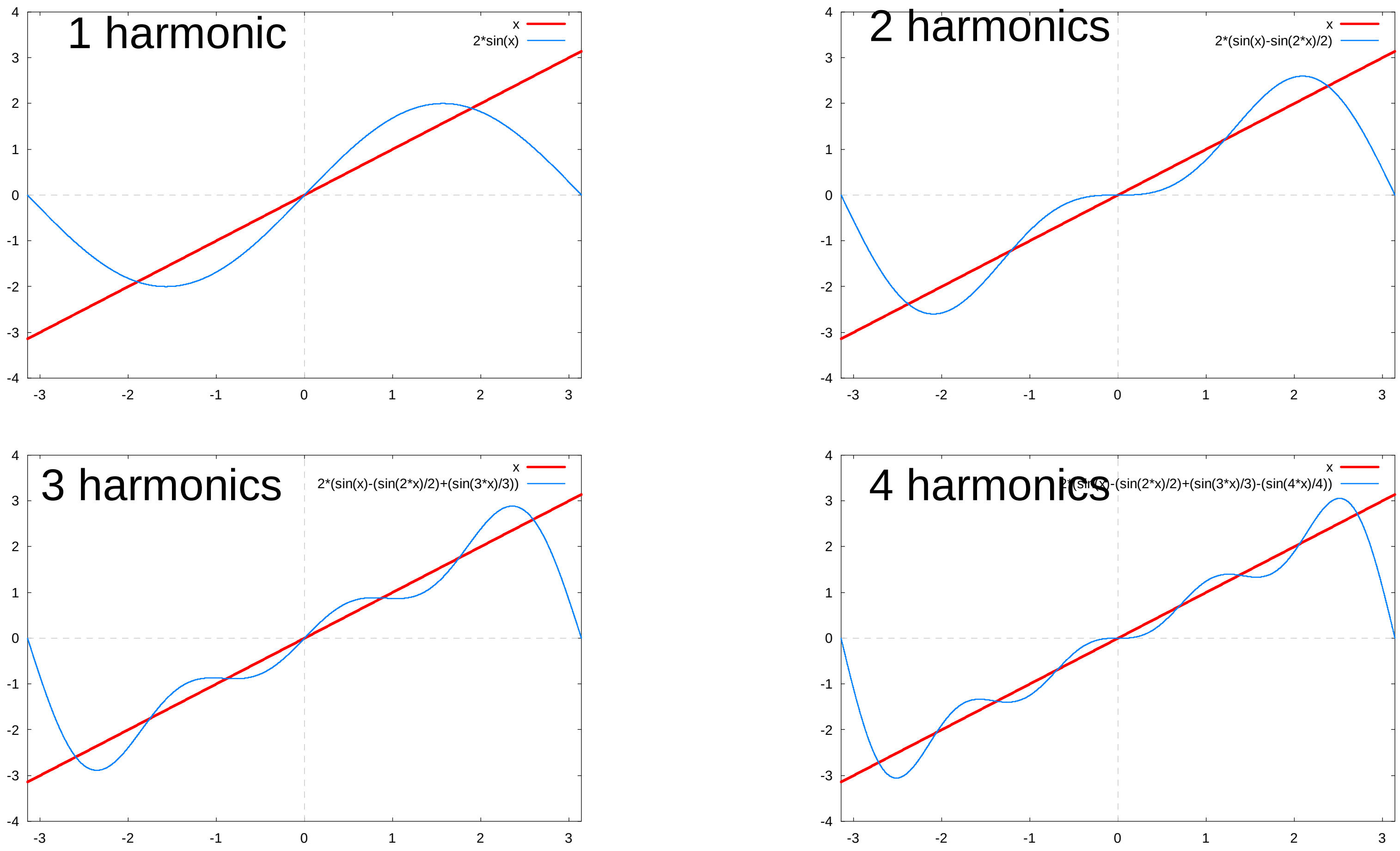
Find the fourier series of the periodic function f(x), where
One period of f(x) is defined as: f(x) = x, $-\pi < x < \pi$



$$T = 2p$$

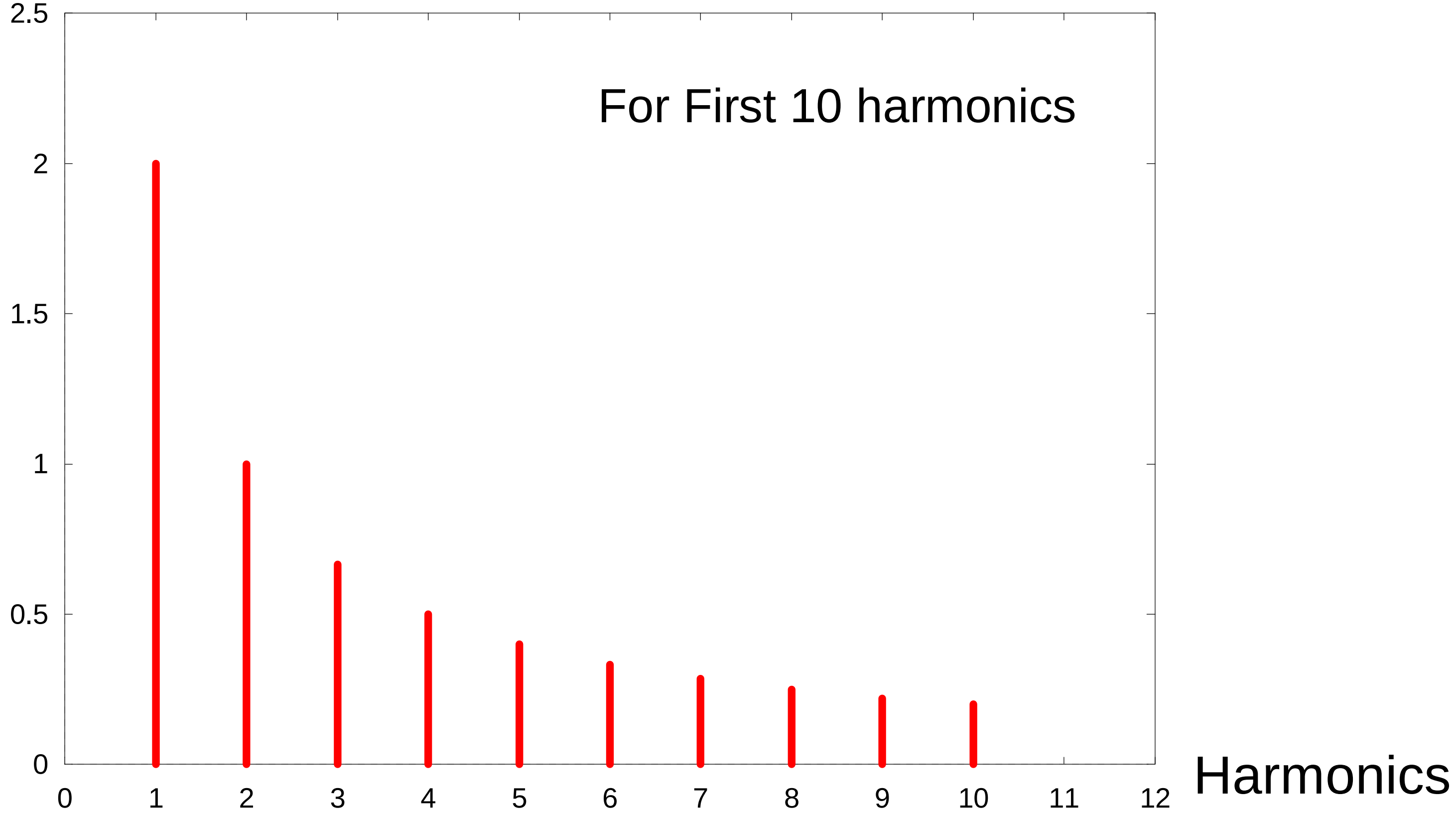
Example: Its Fourier Approximation

Domain: [-π, π]



Example: Frequency Spectrum

Magnitude : $\sqrt{a_n^2 + b_n^2}$



Each harmonic corresponds to a frequency that is multiple of the fundamental frequency

Complex Form of Fourier Series

By substituting Euler's Expresssion into Fouries expansion:

$$e^{jq} = \cos q + j \sin q$$

It can be shown that the following is true:

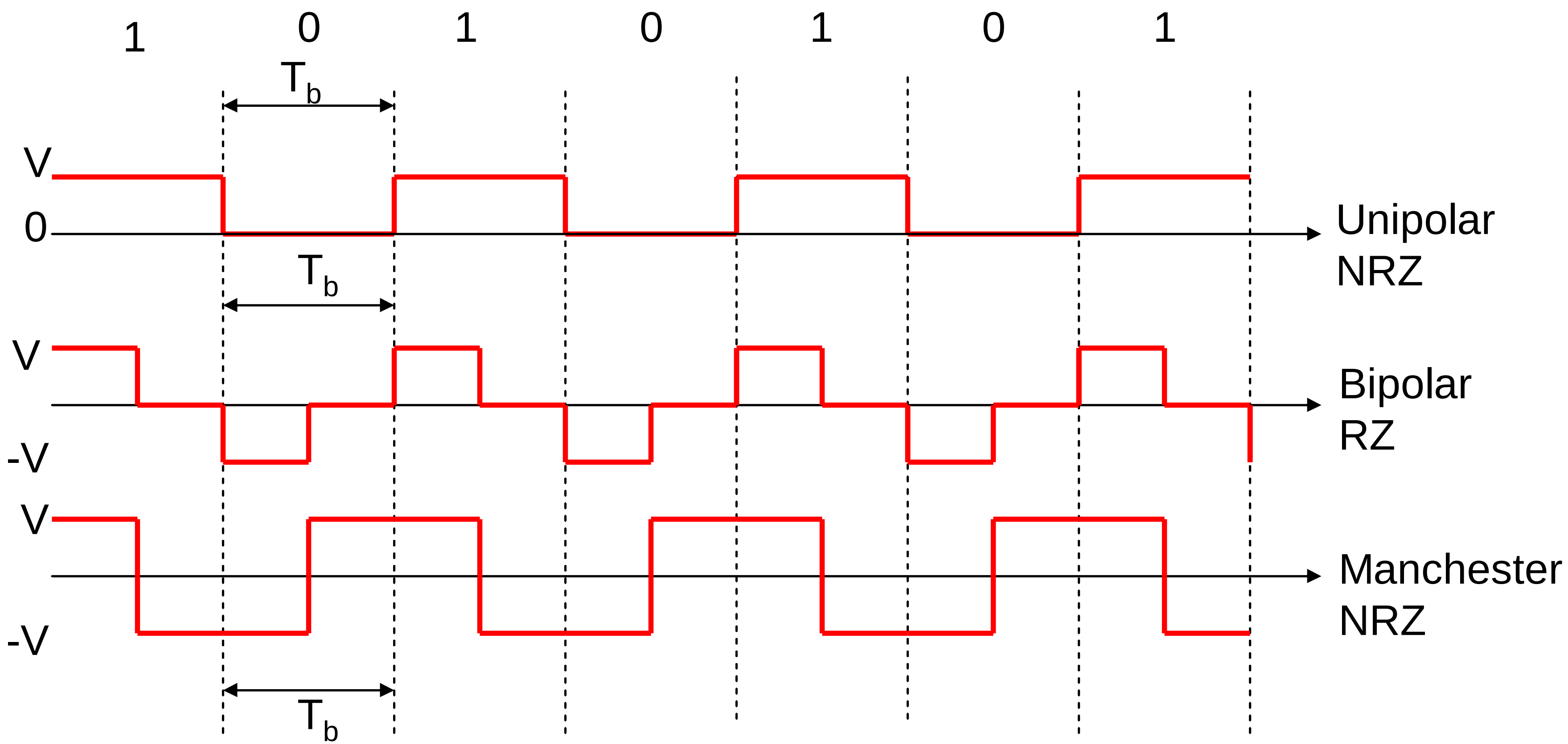
$$F(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnw_T t} dt$$

$$c_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} F(t) e^{-jnw_T t} dt, \quad n = \ldots -2,-1,0,1,2,\ldots.$$

c_n are the complex Fourier coefficients

Digital Modulation - Continues

- Line Coding
  - Base-band signals are represented as line codes



Pulse Shaping Techniques

- Effect of bandlimited channel:
  - Rectangular pulses?
  - Pulses will spread in time
  - Each symbol will interfer succeeding symbols (ISI)
  - Increase the symbol error probability (SER).
- How to mitigate ISI?
  - increase the channel bandwidth (usually infeasible)
  - design pulse shapes carefully (desirable & popular)
- Spectral shaping is usually done through baseband or IF processing.
- Objectives of pulse shaping techniques:
  - Minimize the ISI
  - Minimize the spectral width of a modulated digital signal

Pulse Shaping Techniques

- Nyquist Criterion for ISI Cancellation
  - rectangular "brick-wall" filter
- Raised Cosine Rolloff Filter
- Gaussian Pulse-shaping Filter

Nyquist Criterion for ISI Cancellation

- Nyquist was the first to solve the problem of overcoming ISI while keeping the transmission bandwidth low.
- To nullify the effect of ISI, the overall response of the communication system (including transmitter, channel, and receiver) should be designed as follows:

$$h_{eff}(nT_s) = \begin{cases} K, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

- Rectangular "brick-wall" filter (ideal low-pass filter)
  - Minimize the spectral width (preferable)
  - Strong side lobes in time domain (sensitive to timing error)

$$h_{eff}(t) = \frac{\sin\left(p\, t/T_s\right)}{p\, t/T_s} \quad \Rightarrow \quad H_{eff}(f) = \frac{1}{f_s}\Pi\left(\frac{f}{f_s}\right)$$

Nyquist Criterion for ISI Cancellation

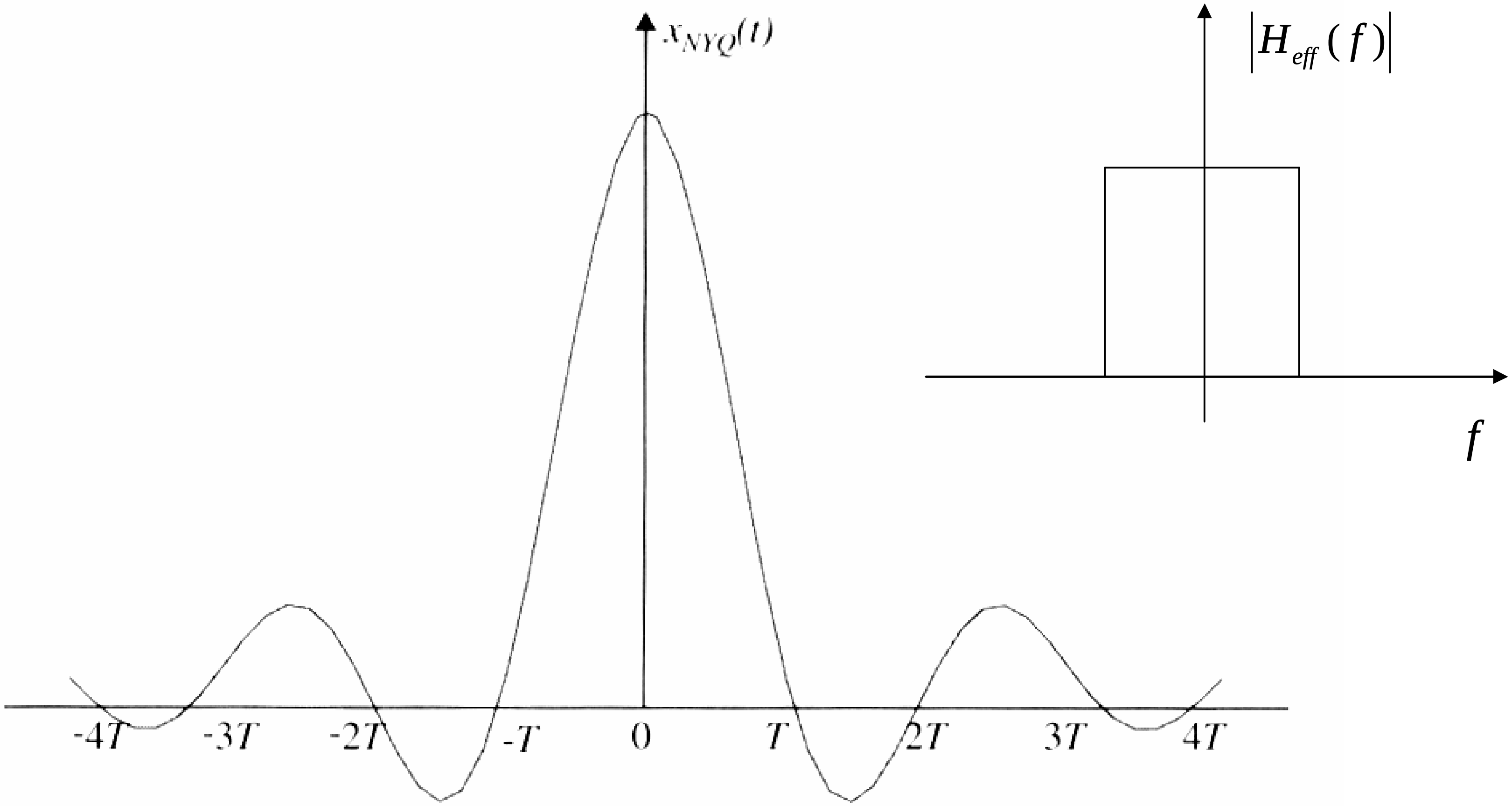


Figure 6.15 Nyquist ideal pulse shape for zero intersymbol interference.

Raised Cosine Rolloff Filter

- The most popular pulse shaping filter used in mobile communications.
- Transfer function

$$H_{RC}(f) = \begin{cases} 1 & 0 \leq |f| \leq \frac{(1-\alpha)}{2T_s} \\ \frac{1}{2}\left[1 + \cos\left[\frac{\pi(|f| \cdot 2T_s - 1 + \alpha)}{2\alpha}\right]\right] & \frac{(1-\alpha)}{2T_s} \leq |f| \leq \frac{(1+\alpha)}{2T_s} \\ 0 & |f| > \frac{(1+\alpha)}{2T_s} \end{cases}$$

α is the rolloff factor which ranges between 0 and 1.

Raised Cosine Spectrum

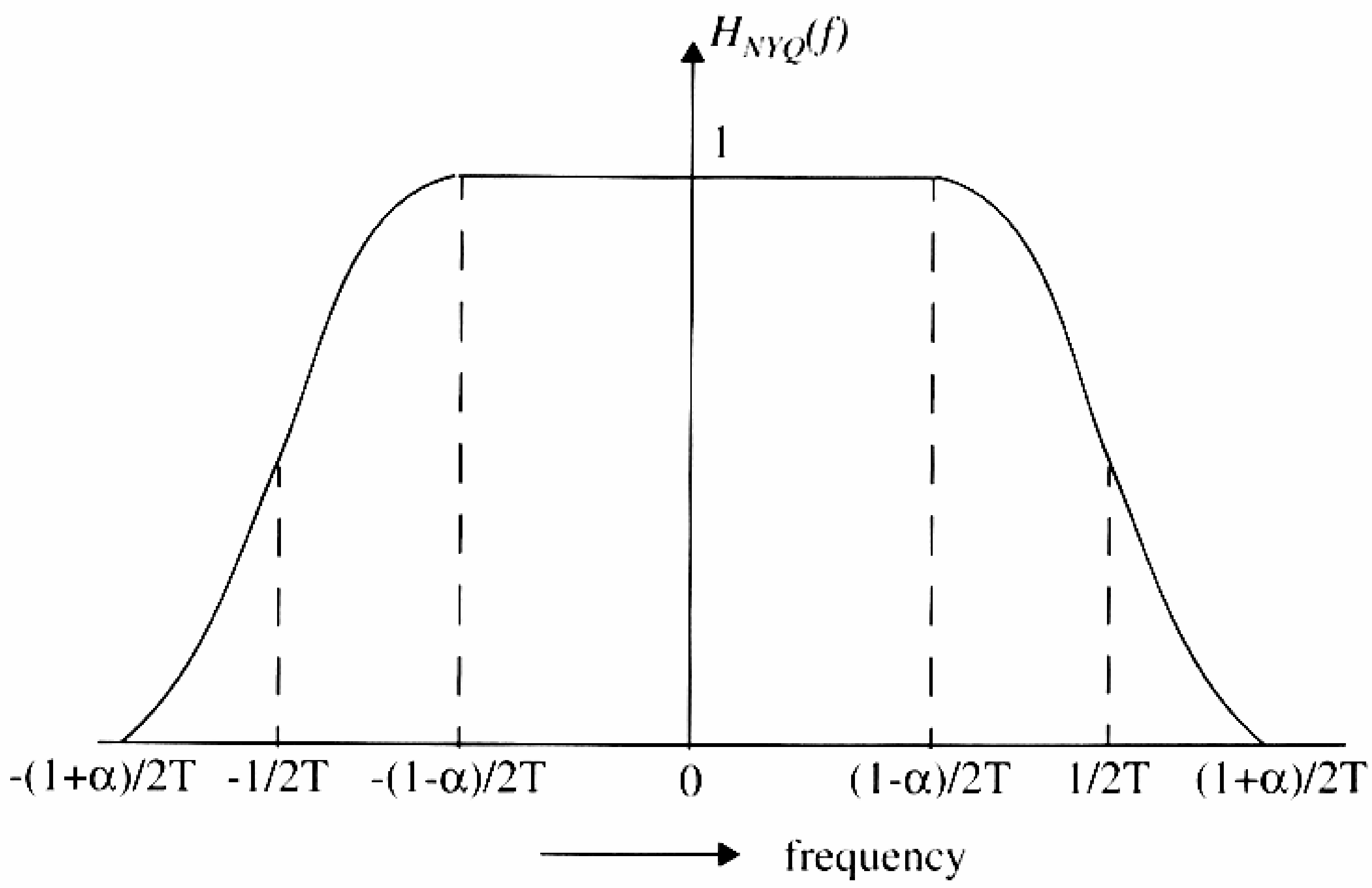


Figure 6.16 Transfer function of a Nyquist pulse-shaping filter at baseband.

Raised Cosine Rolloff Filter

- Impulse response of the cosine rolloff filter

$$h_{RC}(t) = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\pi t} \cdot \frac{\cos\left(\frac{\pi \alpha t}{T_s}\right)}{1 - \left(\frac{4\alpha t}{2T_s}\right)^2}$$

- A raised cosine filter belongs to the class of filters which satisfy the Nyquist criterion.
- As the rolloff factor α increases, the bandwidth of the filter also increases, and the time sidelobe levels decrease in adjacent symbol slots. This implies that increasing α decreases the sensitivity to timing jitter, but increases the occupied bandwidth.

Raised Cosine pulses

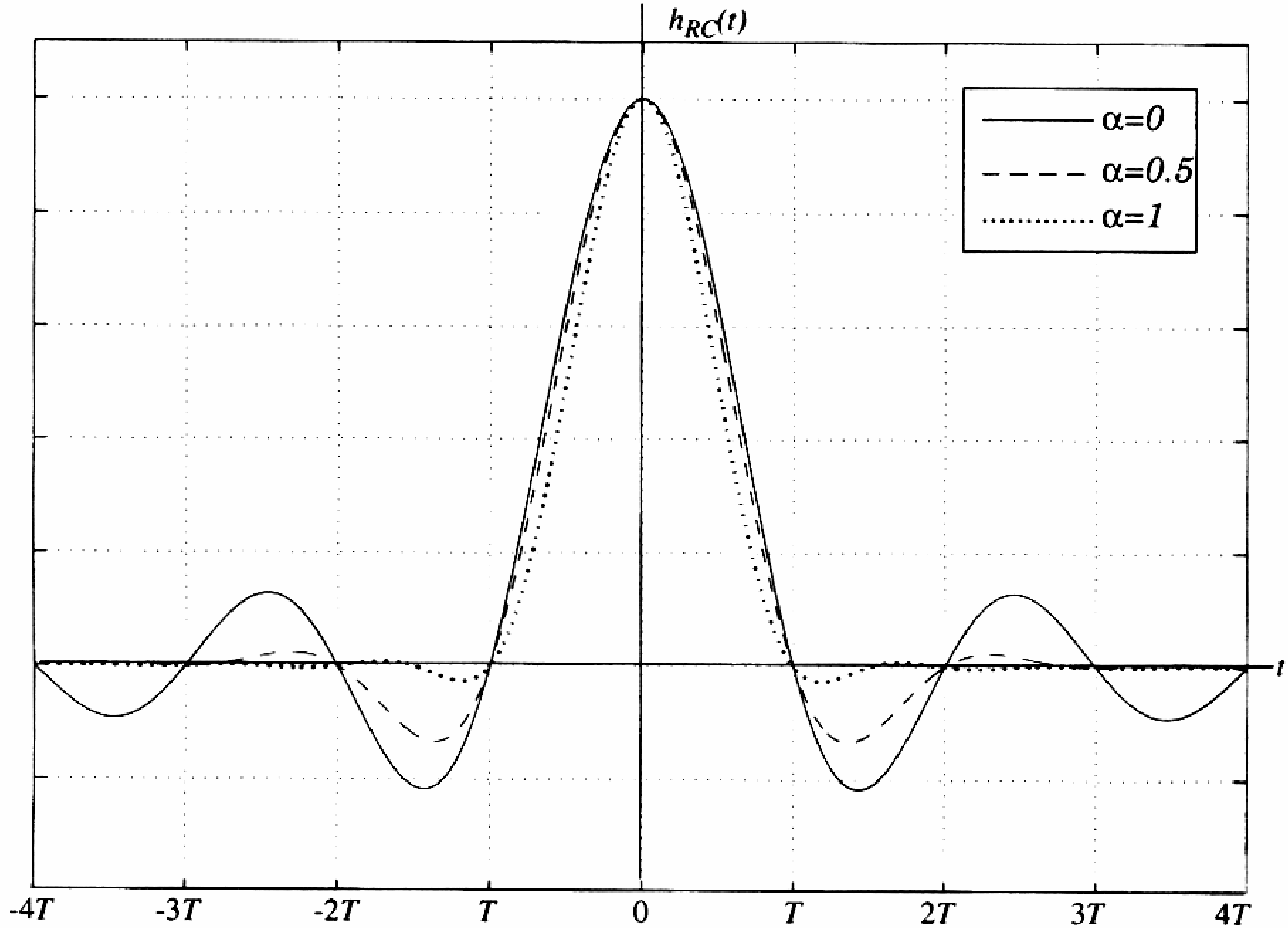


Figure 6.18 Impulse response of a raised cosine rolloff filter at baseband.

Spectrum of Raised Cosine pulse

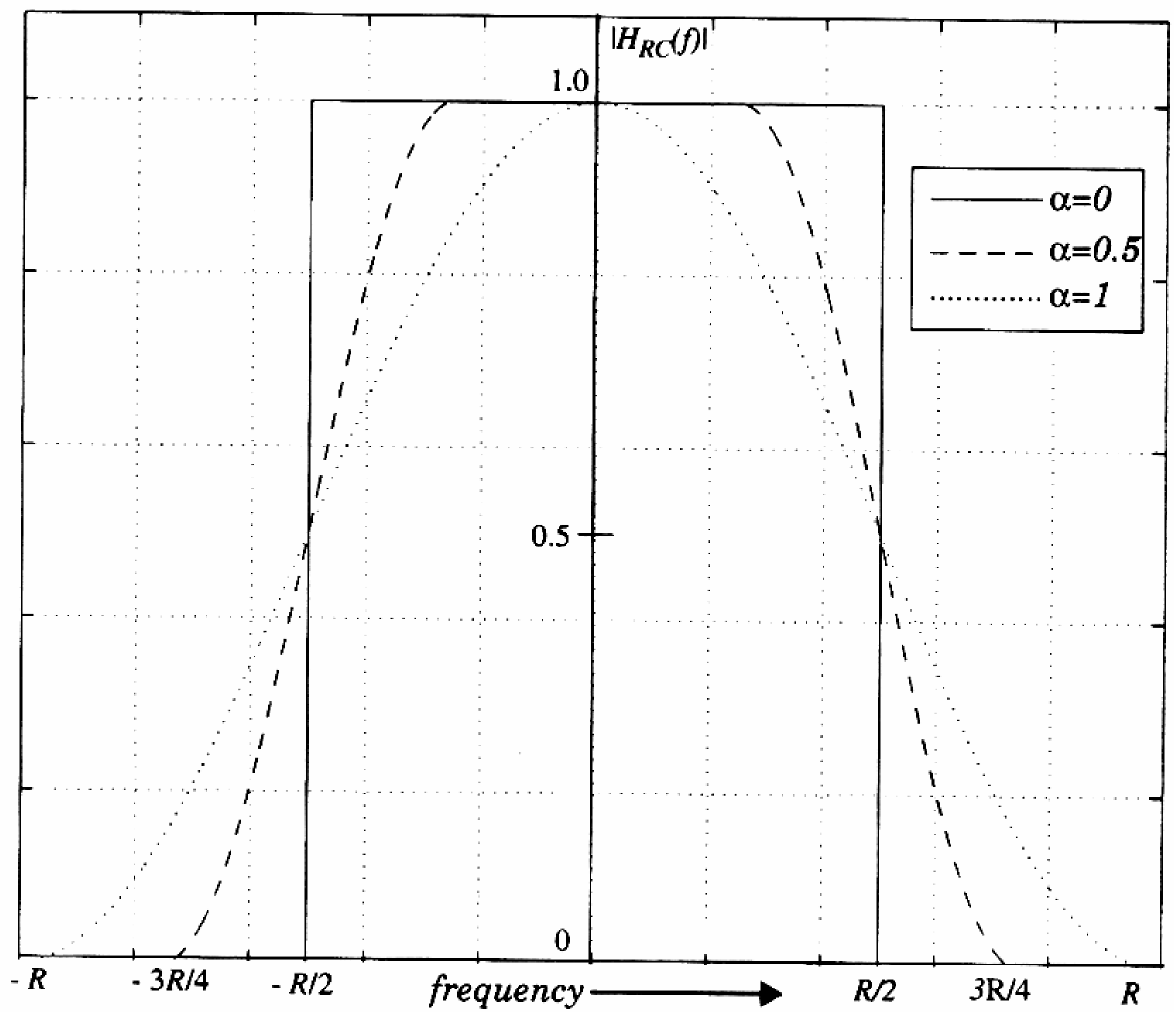


Figure 6.17 Magnitude transfer function of a raised cosine filter at baseband.

Gaussian Pulse-shaping Filter

- Non-Nyquist techniques for pulse shaping.
- Particularly effective when used in conjunction with Minimum Shift Keying (MSK) modulation.
- Transfer function $$H_G(f) = \exp(-\alpha^2 f^2)$$

The parameter α is related to B, the 3-dB bandwidth of the baseband gaussian shaping filter. As α increases, the spectral occupancy of the Gaussian filter decreases and time dispersion of the applied signal increases

$$\alpha = \frac{\sqrt{\ln 2}}{\sqrt{2}B} = \frac{0.5887}{B}$$

Gaussian Pulse-shaping Filter

- The impulse response of the Gaussian filter

$$h_G(t) = \frac{\sqrt{\pi}}{\alpha}\exp\left(-\frac{\pi^2}{\alpha^2}t^2\right)$$

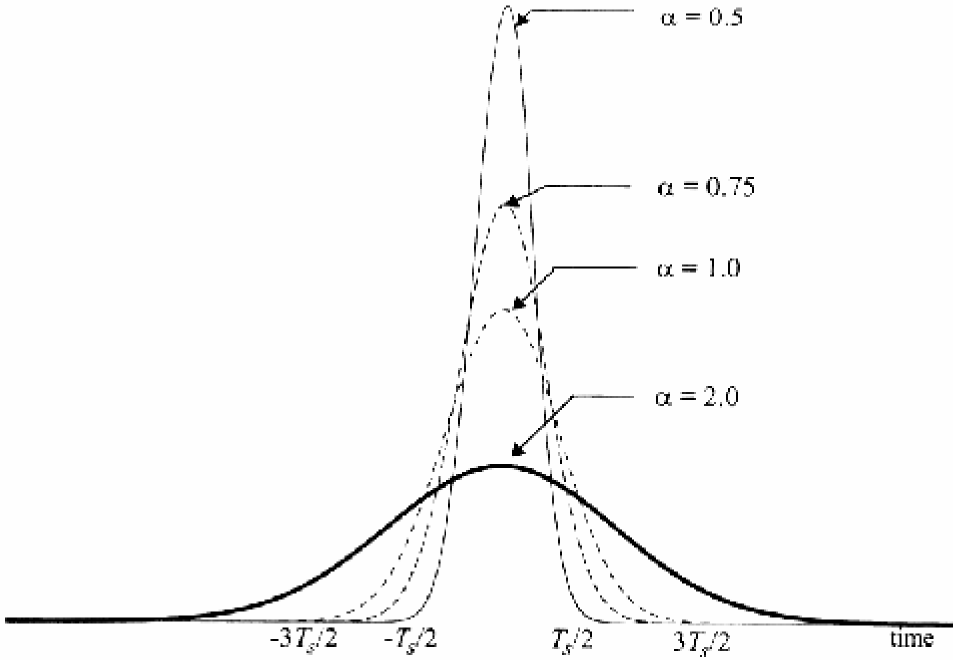


Figure 6.20 Impulse response of a Gaussian pulse-shaping filter.

Geometric Representation of Modulation Signal

- Digital Modulation involves
  - Choosing a particular signal waveform for transmission for a particular symbol or signal
  - For M possible signals, the set of all signal waveforms are:

$$S = \{s_1(t), s_2(t), \ldots, s_M(t)\}$$

- For binary modulation, each bit is mapped to a signal from a set of signal set S that has two signals
- We can view the elements of S as points in vector space

Geometric Representation of Modulation Signal

- Vector space
  - We can represented the elements of S as linear combination of <u>basis signals.</u>
  - The number of basis signals are the dimension of the vector space.
  - Basis signals are orthogonal to each-other.
  - Each basis is normalized to have unit energy:

$$E = \int_{-\infty}^{\infty} f_i^2(t)dt = 1$$

$f_i(t)$ is the i^{th} basis signal.

Example

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2pf_c t) \quad 0 \le t \le T_b$$

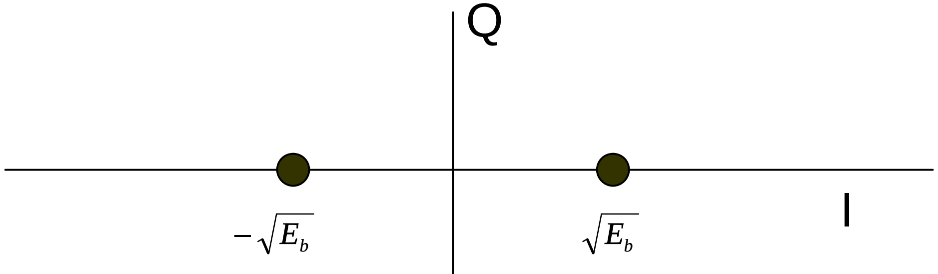
$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}}\cos(2pf_c t) \quad 0 \le t \le T_b$$

Two signal waveforms to be used for transmission

$$f_1(t) = \sqrt{\frac{2}{T_b}}\cos(2pf_c t)$$

The basis signal

$$S = \left\{\sqrt{E_b}\,f_1(t), -\sqrt{E_b}\,f_1(t)\right\}$$



Constellation Diagram
Dimension = 1

Constellation Diagram

- Properties of Modulation Scheme can be inferred from Constellation Diagram
  - Bandwidth occupied by the modulation increases as the dimension of the modulated signal increases
  - Bandwidth occupied by the modulation decreases as the signal_points per dimension increases (getting more dense)
  - Probability of bit error is proportional to the distance between the closest points in the constellation.
    - Bit error decreases as the distance increases (sparse).

Linear Modulation Techniques

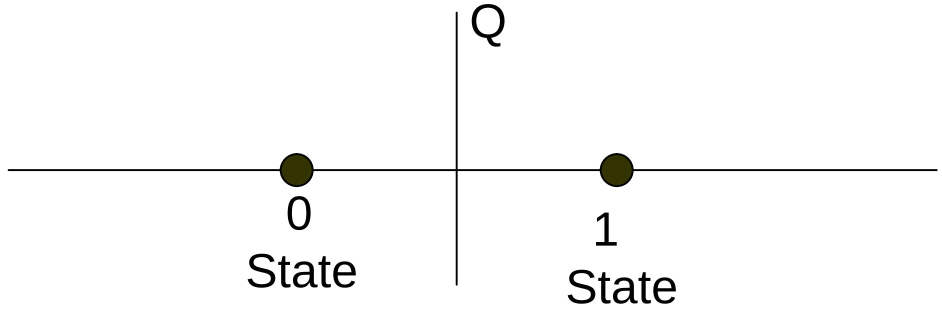
- Classify digital modulation techniques as:
  - Linear
    - The amplitude of the transmitted signal varies linearly with the modulating digital signal, m(t).
    - They usually do not have constant envelope.
    - More spectral efficient.
    - Poor power efficiency
    - Examples: QPSK,OQPSK.
  - Non-linear

Binary Phase Shift Keying

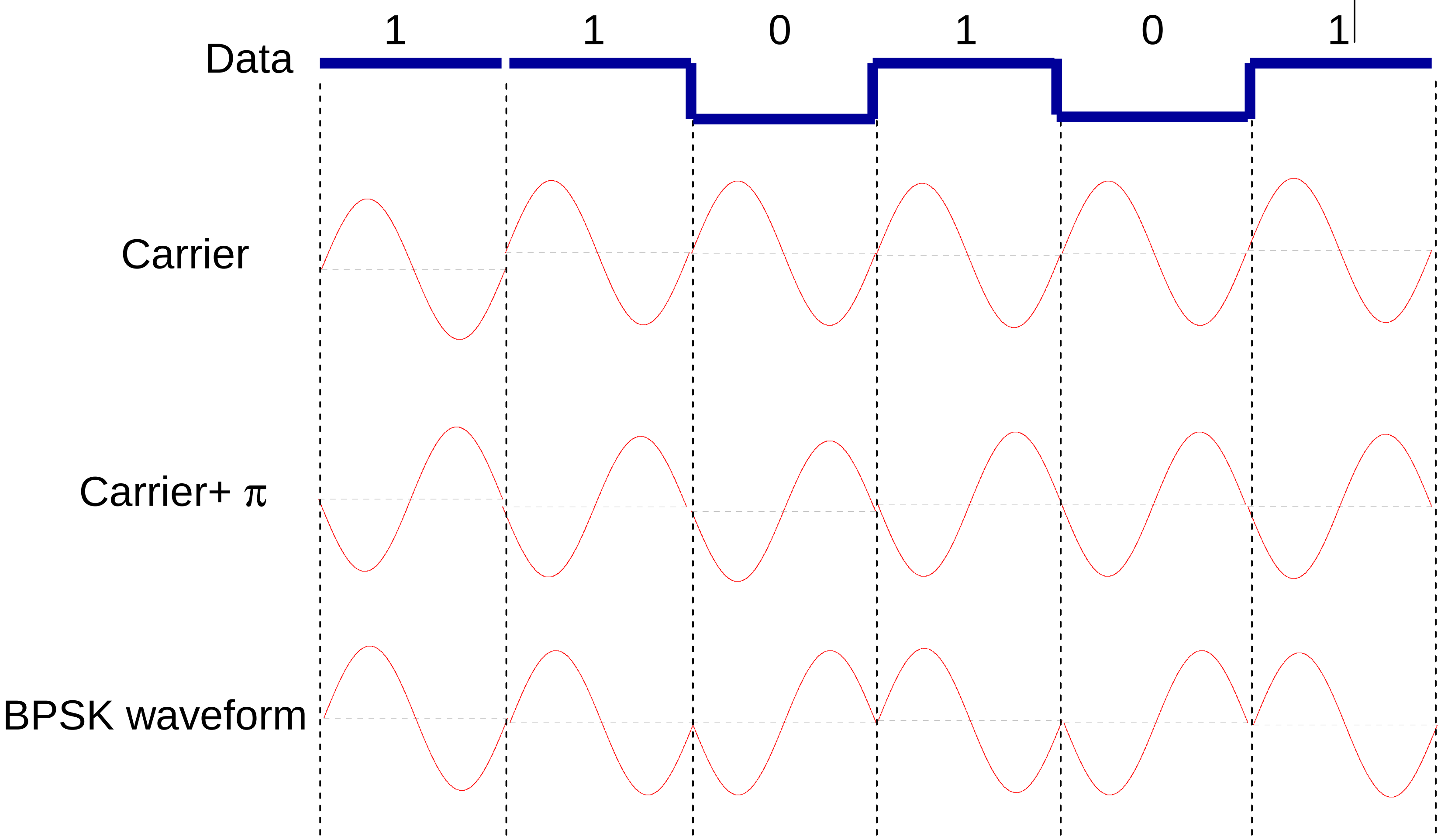
- Use alternative sine wave phase to encode bits
  - Phases are separated by 180 degrees.
  - Simple to implement, inefficient use of bandwidth.
  - Very robust, used extensively in satellite communication.

$$s_1(t) = A_c \cos(2p\, f_c t + q_c) \qquad \text{binary 1}$$

$$s_2(t) = A_c \cos(2p\, f_c t + q_c + p) \quad \text{binary 0}$$

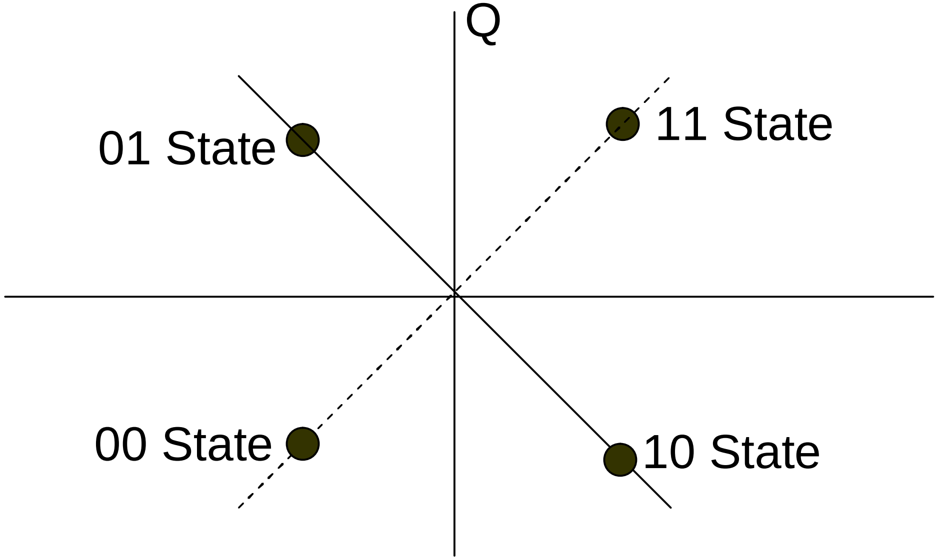


BPSK Example



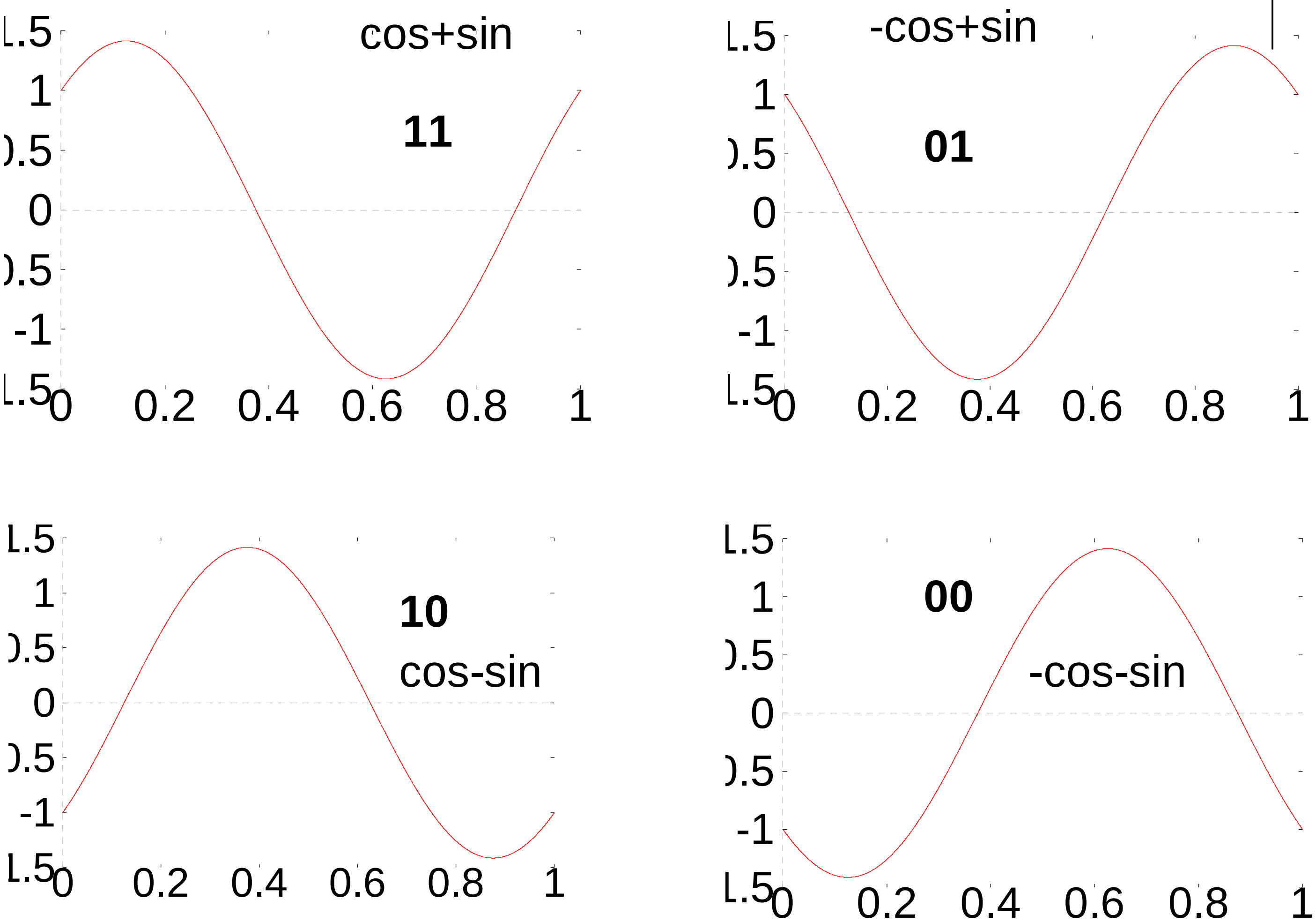
Quadrature Phase Shift Keying

- Multilevel Modulation Technique: 2 bits per symbol
- More spectrally efficient, more complex receiver.
- Two times more bandwidth efficient than BPSK



Phase of Carrier: $\pi/4$, $2\pi/4$, $5\pi/4$, $7\pi/4$

4 different waveforms



Constant Envelope Modulation

- Amplitude of the carrier is constant, regardless of the variation in the modulating signal
  - Better immunity to fluctuations due to fading.
  - Better random noise immunity
  - Power efficient
- They occupy larger bandwidth
- Examples: FSK, MSK, GMSK

Frequency Shift Keying (FSK)

- The frequency of the carrier is changed according to the message state (high (1) or low (0)).

$$s_1(t) = A\cos(2pf_c + 2p\Delta f)t \quad 0 \leq \text{t} \leq \text{T}_\text{b} \text{ (bit} = 1)$$

$$s_2(t) = A\cos(2pf_c - 2p\Delta f)t \quad 0 \leq \text{t} \leq \text{T}_\text{b} \text{ (bit} = 0)$$

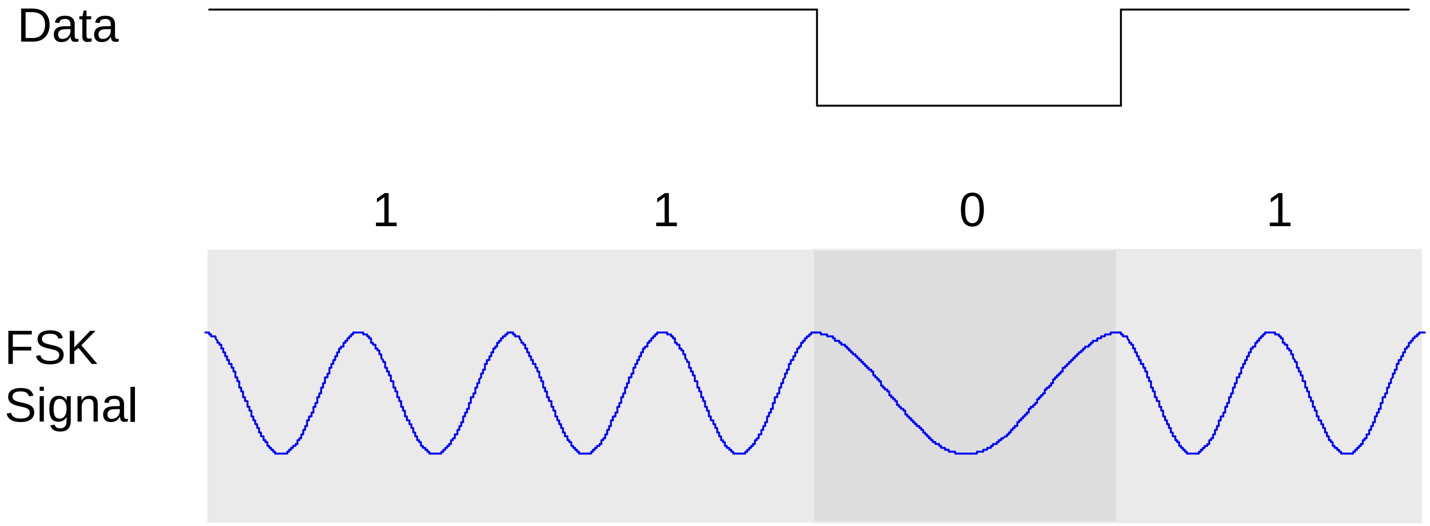
Continues FSK

$$s(t) = A\cos(2pf_c + q(t))$$

$$s(t) = A\cos(2pf_c t + 2pk_f \int_{-\infty}^{t} m(x)dx)$$

Integral of m(x) is continues.

FSK Example



Combined Linear and Constant Envelope Modulation Techniques

- Modem modulation techniques exploit the fact that digital baseband data may be sent by varying both the envelope and phase (or frequency) of an RF carrier.
- M-ary signaling: two or more bits are grouped together to form symbols, where $M=2^n$ (n is an integer).
  - MASK
  - MPSK
  - MFSK
  - MQAM
- M-ary modulation schemes achieve better bandwidth efficiency at the expense of power efficiency.

M-ary Phase Shift Keying (MPSK)

- Modulated signal

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2p\, f_c t + \frac{M}{2p}\left(i-1\right) \right), 0 \le t \le T_s, i = 1, 2, \ldots, M$$

- Probability of symbol error in an AWGN channel

$$P_e \le 2Q\left(\sqrt{\frac{E_s}{N_0}} \sin \frac{p}{M} \right) = 2Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}} \sin \frac{p}{M} \right)$$

8-PSK Signal Constellation

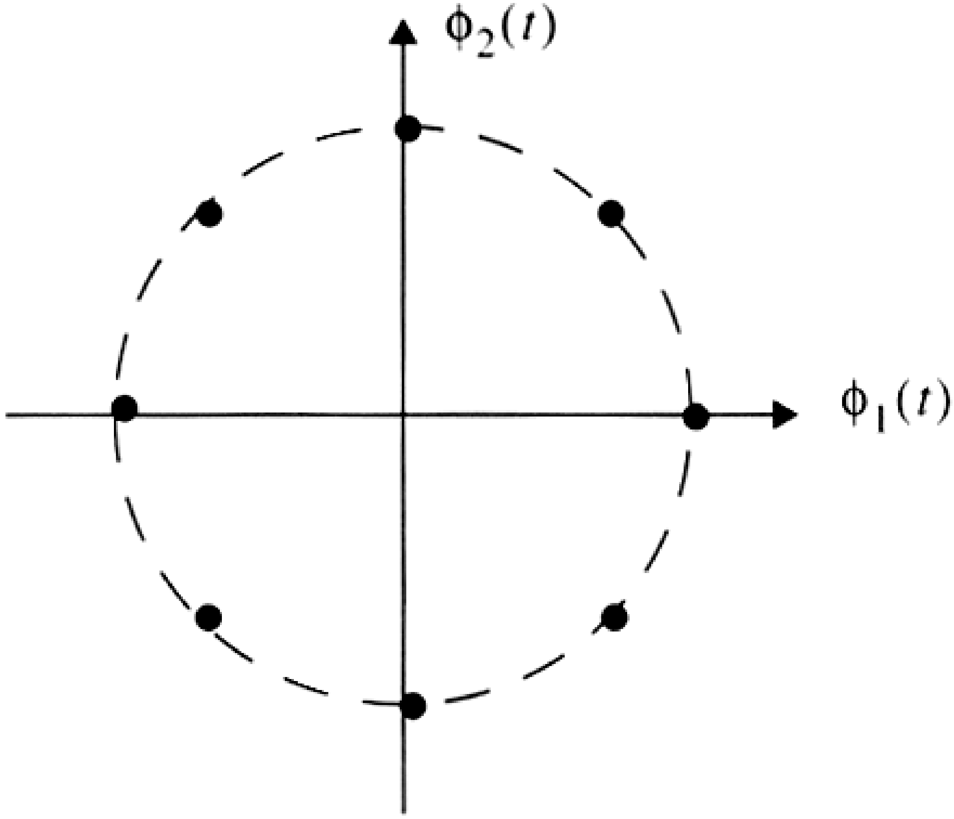


Figure 6.45 Constellation diagram of an M-ary PSK system ($M = 8$).

M-ary Quadrature Amplitude Modulation (QAM)

- Modulated signal

$$s_i(t) = \sqrt{\frac{2E_{\min}}{T_s}} \cdot a_i \cos\left(2p\ f_c t\right) + \sqrt{\frac{2E_{\min}}{T_s}} \cdot b_i \sin\left(2p\ f_c t\right)$$

$$, 0 \le t \le T_s, i = 1, 2, ..., M$$

- Probability of symbol error in an AWGN channel

$$P_e \cong 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_{\min}}{N_0}}\right) = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_{av}}{(M-1)N_0}}\right)$$

16-QAM Signal Constellation

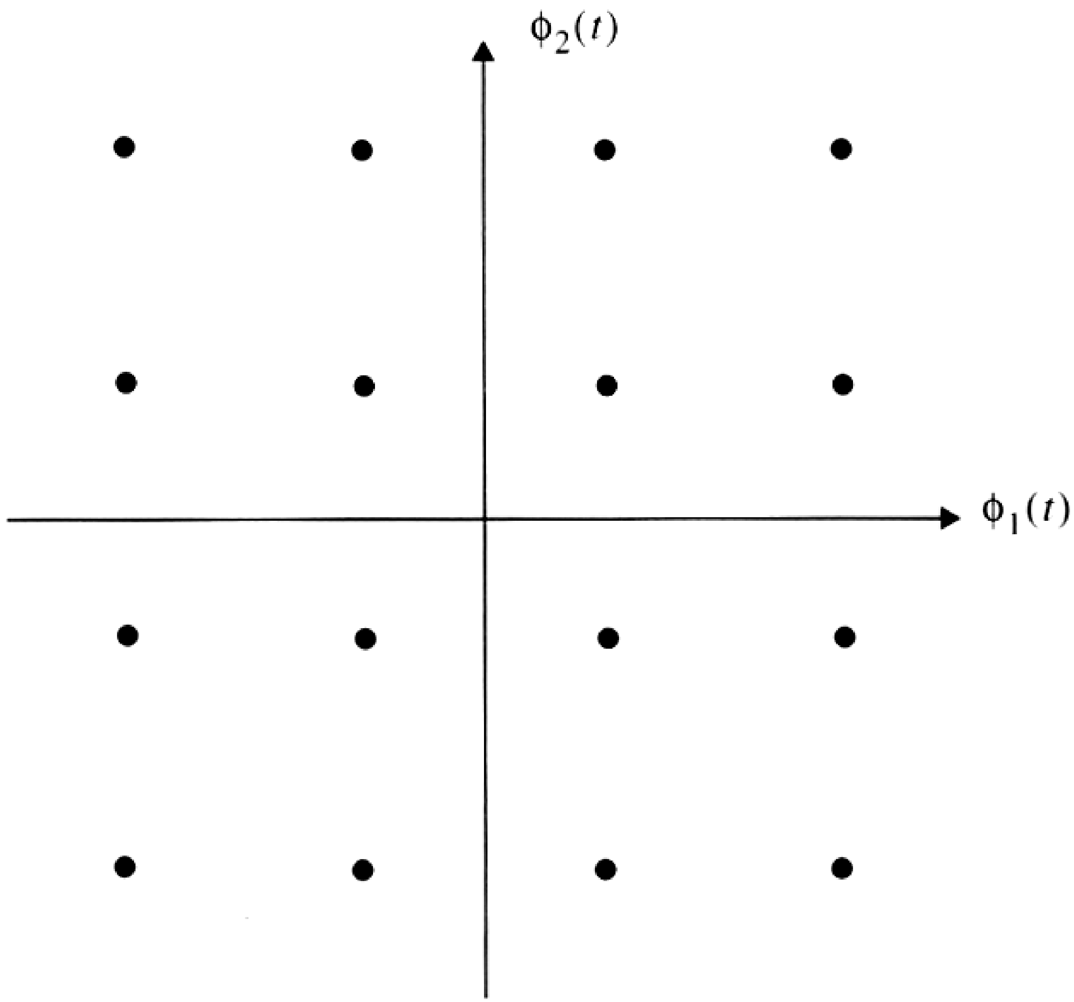


Figure 6.47 Constellation diagram of an M-ary QAM ($M = 16$) signal set.