Case 2

Osmolarity, Osmotic Pressure, and Osmosis

The information shown in Table 1–2 pertains to six different solutions.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Solute</th>
<th>Concentration</th>
<th>g</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Urea</td>
<td>1 mmol/L</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>NaCl</td>
<td>1 mmol/L</td>
<td>1.85</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>NaCl</td>
<td>2 mmol/L</td>
<td>1.85</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>KCl</td>
<td>1 mmol/L</td>
<td>1.85</td>
<td>0.4</td>
</tr>
<tr>
<td>5</td>
<td>Sucrose</td>
<td>1 mmol/L</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>6</td>
<td>Albumin</td>
<td>1 mmol/L</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

g, osmotic coefficient; σ, reflection coefficient.

**Questions**

1. What is osmolarity, and how is it calculated?

2. What is osmosis? What is the driving force for osmosis?

3. What is osmotic pressure, and how is it calculated? What is effective osmotic pressure, and how is it calculated?

4. Calculate the osmolarity and effective osmotic pressure of each solution listed in Table 1–2 at 37°C. For 37°C, RT = 25.45 L-atm/mol, or 0.0245 L-atm/mmol.

5. Which, if any, of the solutions are isosmotic?

6. Which solution is hyperosmotic with respect to all of the other solutions?

7. Which solution is hypotonic with respect to all of the other solutions?

8. A semipermeable membrane is placed between Solution 1 and Solution 6. What is the difference in effective osmotic pressure between the two solutions? Draw a diagram that shows how water will flow between the two solutions and how the volume of each solution will change with time.

9. If the hydraulic conductance, or filtration coefficient (K), of the membrane in Question 8 is 0.01 mL/min-atm, what is the rate of water flow across the membrane?

10. Mannitol is a large sugar that does not dissociate in solution. A semipermeable membrane separates two solutions of mannitol. One solution has a mannitol concentration of 10 mmol/L, and the other has a mannitol concentration of 1 mmol/L. The filtration coefficient of the membrane is 0.5 mL/min-atm, and water flow across the membrane is measured as 0.1 mL/min. What is the reflection coefficient of mannitol for this membrane?
### Answers and Explanations

1. **Osmolarity** is the concentration of osmotically active particles in a solution. It is calculated as the product of solute concentration (e.g., in mmol/L) times the number of particles per mole in solution (i.e., whether the solute dissociates in solution). The extent of this dissociation is described by an **osmotic coefficient** called “g.” If the solute does not dissociate, g = 1.0. If the solute dissociates into two particles, g = 2.0, and so forth. For example, for solutes such as urea or sucrose, g = 1.0 because these solutes do not dissociate in solution. On the other hand, for NaCl, g = 2.0 because NaCl dissociates into two particles in solution, Na⁺ and Cl⁻. With this last example, it is important to note that Na⁺ and Cl⁻ ions may interact in solution, making g slightly less than the theoretical, ideal value of 2.0.

\[
\text{Osmolarity} = g \times C
\]

Where
- \( g \) = number of particles/mol in solution (in some texts, \( g = n \times \Phi \))
- \( C \) = concentration (e.g., mmol/L)

Two solutions that have the same calculated osmolarity are called **isosmotic**. If the calculated osmolarity of two solutions is different, then the solution with the higher osmolarity is hyperosmotic and the solution with the lower osmolarity is hyposmotic.

2. **Osmosis** is the flow of water between two solutions separated by a semipermeable membrane caused by a difference in solute concentration. The driving force for osmosis is a difference in osmotic pressure caused by the presence of solute. Initially, it may be surprising that the presence of solute can cause a pressure, which is explained as follows. Solute particles in a solution interact with pores in the membrane and, in so doing, lower the hydrostatic pressure of the solution. The higher the solute concentration, the higher the osmotic pressure (see Question 3) and the lower the hydrostatic pressure (because of the interaction of solute with pores in the membrane). Thus, if two solutions have different solute concentrations, their osmotic and hydrostatic pressures are also different; the difference in pressure causes water flow across the membrane (i.e., osmosis).

3. The **osmotic pressure** of a solution is described by the van’t Hoff equation:

\[
\pi = g \times C \times RT
\]

Where
- \( \pi \) = osmotic pressure [atmospheres (atm)]
- \( g \) = number of particles/mol in solution
- \( C \) = concentration (e.g., mmol/L)
- \( R \) = gas constant (0.082 L-atm/mol-K)
- \( T \) = absolute temperature (K)

In words, the van’t Hoff equation states that the osmotic pressure of a solution depends on the concentration of osmotically active solute particles. The concentration of solute particles is converted to a pressure by multiplying it by the gas constant and the absolute temperature.

The concept of “effective” osmotic pressure involves a slight modification of the van’t Hoff equation. Effective osmotic pressure depends on both the concentration of solute particles and the extent to which the solute crosses the membrane. The extent to which a particular solute crosses a particular membrane is expressed by a dimensionless factor called the **reflection coefficient** (\( \sigma \)). The value of the reflection coefficient can vary from 0 to 1.0 (Figure 1–1). When \( \sigma = 1.0 \), the membrane is completely impermeable to the solute; the solute remains in the original solution and exerts its full osmotic pressure. When \( \sigma = 0 \), the mem-
brane is freely permeable to the solute; solute diffuses across the membrane and down its concentration gradient until the concentrations in both solutions are equal. In this case, where $\sigma = 0$, the solutions on either side of the membrane have the same osmotic pressure because they have the same solute concentration; there is no difference in effective osmotic pressure across the membrane, and no osmosis of water occurs. When $\sigma$ is between 0 and 1, the membrane is somewhat permeable to the solute; the effective osmotic pressure lies somewhere between its maximal value and 0.

Thus, to calculate the effective osmotic pressure ($\pi_{\text{eff}}$), the van't Hoff equation for osmotic pressure is modified by the value for $\sigma$, as follows:

$$\pi_{\text{eff}} = g C \sigma RT$$

where

- $\pi_{\text{eff}} =$ effective osmotic pressure (atm)
- $g =$ number of particles/mol in solution
- $C =$ concentration (e.g., mmol/L)
- $R =$ gas constant (0.082 L-atm/mol-K)
- $T =$ absolute temperature (K)
- $\sigma =$ reflection coefficient (no units; varies from 0 to 1)

Isotonic solutions have the same effective osmotic pressure. When isotonic solutions are placed on either side of a semipermeable membrane, there is no difference in effective osmotic pressure across the membrane, no driving force for osmosis, and no water flow.

If two solutions have different effective osmotic pressures, then the one with the higher effective osmotic pressure is hypertonic, and the one with the lower effective osmotic pressure is hypotonic. If these solutions are placed on either side of a semipermeable membrane, then an osmotic pressure difference is present. This osmotic pressure difference is the driving force for water flow. Water flows from the hypotonic solution (with the lower effective osmotic pressure) into the hypertonic solution (with the higher effective osmotic pressure).
4. See Table 1-3.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Osmolarity (mOsm/L)</th>
<th>Effective Osmotic Pressure (atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.85</td>
<td>0.0227</td>
</tr>
<tr>
<td>3</td>
<td>3.7</td>
<td>0.0453</td>
</tr>
<tr>
<td>4</td>
<td>1.85</td>
<td>0.0181</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0.0196</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.0245</td>
</tr>
</tbody>
</table>

5. Solutions with the same calculated osmolarity are isosmotic. Therefore, Solutions 1, 5, and 6 are isosmotic with respect to each other. Solutions 2 and 4 are isosmotic with respect to each other.

6. Solution 3 has the highest calculated osmolarity. Therefore, it is hyperosmotic with respect to the other solutions.

7. According to our calculations, Solution 1 is hypotonic with respect to the other solutions because it has the lowest effective osmotic pressure (zero). But why zero? Shouldn't the urea particles in Solution 1 exert some osmotic pressure? The answer lies in the reflection coefficient of urea, which is zero: because the membrane is freely permeable to urea, urea diffuses down its concentration gradient until the concentrations of urea on both sides of the membrane are equal. At this point of equal concentration, urea exerts no "effective" osmotic pressure.

8. Solution 1 is 1 mmol/L urea, with an osmolarity of 1 mOsm/L and an effective osmotic pressure of 0. Solution 6 is 1 mmol/L albumin, with an osmolarity of 1 mOsm/L and an effective osmotic pressure of 0.0245 atm. According to the previous discussion, these two solutions are isosmotic because they have the same osmolarity. However, they are not isotonic because they have different effective osmotic pressures. Solution 1 (urea) has the lower effective osmotic pressure and is hypotonic. Solution 6 (albumin) has the higher effective osmotic pressure and is hypertonic. The effective osmotic pressure difference ($\Delta \pi_{\text{eff}}$) is the difference between the effective osmotic pressure of Solution 6 and that of Solution 1:

$$\Delta \pi_{\text{eff}} = \pi_{\text{eff}} (\text{Solution 6}) - \pi_{\text{eff}} (\text{Solution 1})$$

$$= 0.0245 \text{ atm} - 0 \text{ atm}$$

$$= 0.0245 \text{ atm}$$

If the two solutions are separated by a semipermeable membrane, water flows by osmosis from the hypotonic urea solution into the hypertonic albumin solution. With time, as a result of this water flow, the volume of the urea solution decreases and the volume of the albumin solution increases, as shown in Figure 1-2.
9. Osmotic water flow across a membrane is the product of the osmotic driving force ($\Delta \pi_{\text{eff}}$) and the water permeability of the membrane, which is called the hydraulic conductance, or filtration coefficient ($K_f$). In this question, $K_f$ is given as 0.01 mL/min-atm, and $\Delta \pi_{\text{eff}}$ was calculated in Question 8 as 0.0245 atm.

Water flow = $K_f \times \Delta \pi_{\text{eff}}$
= 0.01 mL/min-atm $\times$ 0.0245 atm
= 0.000245 mL/min

10. This question is approached by using the relationship between water flow, hydraulic conductance ($K_f$), and difference in effective osmotic pressure that was introduced in Question 9. For each mannitol solution, $\pi_{\text{eff}} = \sigma g C RT$. Therefore, the difference in effective osmotic pressure between the two mannitol solutions ($\Delta \pi_{\text{eff}}$) is:

$\Delta \pi_{\text{eff}} = \sigma g \Delta C \times RT$
$\Delta \pi_{\text{eff}} = \sigma \times 1 \times (10 \text{ mmol/L} - 1 \text{ mmol/L}) \times 0.0245 \text{ L-atm/mmol}$
= $\sigma \times 0.2205 \text{ atm}$

Now, substituting this value for $\Delta \pi_{\text{eff}}$ into the expression for water flow:

Water flow = $K_f \times \Delta \pi_{\text{eff}}$
= $K_f \times \sigma \times 0.2205 \text{ atm}$
Rearranging, substituting the value for water flow (0.1 mL/min), and solving for $\sigma$:

$$\sigma = \frac{0.1 \text{ mL}}{\text{min}} \times \frac{\text{min - atm}}{0.5 \text{ mL}} \times \frac{0.2205 \text{ atm}}{0.2205 \text{ atm}}$$

$$= 0.91$$

**Key topics**

- Effective osmotic pressure ($\pi_{en}$)
- Filtration coefficient ($K_d$)
- Hyperosmotic
- Hypertonic
- Hyposmotic
- Hypotonic
- Isosmotic
- Isotonic
- Osmolarity
- Osmosis
- Osmotic coefficient
- Osmotic pressure ($\pi$)
- Reflection coefficient ($\sigma$)
- Van't Hoff equation